

Englewood on the Palisades Charter School
Grade 6 Mathematics Curriculum
Aligned With the 2023 New Jersey Student Learning Standards

The Vision for Mathematics Education in New Jersey

A New Jersey education in Mathematics builds quantitatively and analytically literate citizens prepared to meet the demands of college and career, and to engage productively in an information-driven society. All students will have access to a high-quality mathematics education that fosters a population that:

- leverages data in decision-making and as a lens for discussing, analyzing, and responding to practical questions.
- persists to make sense of and model problems arising in everyday life, society, and the workplace.
- thinks critically and strategically to assess quantitative relationships and to solutions to complex problems.
- employs precise reasoning and constructs viable arguments to deduce conclusions, recognize false statements and assess peers' reasoning.
- interprets, evaluates and critiques the mathematics embedded in social, scientific and commercial systems, as well as the claims made in the private and public sectors.
- communicates precisely when conveying, representing, and justifying both qualitative and quantitative perspectives.

(NJ DOE, 2024)

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

Standard	Mathematically Proficient Students ...
1. Make sense of problems and persevere in solving them	start by explaining to themselves the meaning of a problem and looking for entry points to its solution. check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?”
2. Reason abstractly and quantitatively	make sense of quantities and their relationships in problem situations.
3. Construct viable arguments and critique the reasoning of others	understand and use stated assumptions, definitions, and previously established results in constructing arguments.
4. Model with mathematics	apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later.
5. Use appropriate tools strategically	are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations.
6. Attend to precision	try to communicate precisely to others.
7. Look for and make use of structure	look closely to discern a pattern or structure
8. Look for and express regularity in repeated reasoning	notice if calculations are repeated, and look both for general methods and for shortcuts.

(NJ DOE, 2024)

New Jersey Legislative Statutes and Administrative Code Mathematics

Amistad Law: *N.J.S.A. 18A 52:16A-88*

[Radical Math](#): includes lessons on topics for social justice in mathematics
[African American Mathematician Bios](#)

Holocaust Law: *N.J.S.A. 18A:35-28*

<https://www.bbc.com/future/article/20191031-hilda-geiringer-mathematician-who-fled-the-nazis>

LGBT and Disabilities Law: *N.J.S.A. 18A:35-4.35*

<https://prideinstem.org/>
[LGBT+ Mathematician Bios](#)
<https://www.ngpf.org/blog/math/math-monday-celebrating-disabled-mathematicians/>
[Mathematicians with Disabilities](#)

Diversity & Inclusion: *N.J.S.A. 18A:35-4.36a*

<http://www.ams.org/about-us/diversity>
<https://mathematicallygiftedandblack.com/>

Standards in Action: *Climate Change*

New Jersey became the first state in the nation to include climate change across content areas with the adoption of the 2020 New Jersey Student Learning Standards (NJSLs). The goal of inclusion of climate change education implementation is to foster

generations of New Jersey students that can analyze, question, interpret, to think independently, and bring critical deduction to fulfill, and to lead in jobs created by burgeoning industries of the future green economy.

Suggestions for how to incorporate climate change examples into math instruction will be noted in **green text** throughout the units.

Math Curriculum

Grade Six

This sixth grade math curriculum aligns with the 2023 New Jersey Student Learning Standards (NJSLS). Instruction in grade six focuses broadly on four critical areas:

- connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems;
- completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers;
- writing, interpreting, and using expressions and equations; and
- developing understanding of statistical thinking.

Sixth graders also build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume.

Domains and Cluster Areas

- **Ratio and Proportional Relationships**
 - Understand ratio concepts and use ratio reasoning to solve problems.
- **The Number System**
 - Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

- Compute fluently with multi-digit numbers and find common factors and multiples.
- Apply and extend previous understandings of numbers to the system of rational numbers.
- **Expressions and Equations**
 - Apply and extend previous understandings of arithmetic to algebraic expressions.
 - Reason about and solve one-variable equations and inequalities.
 - Represent and analyze quantitative relationships between dependent and independent variables.
- **Geometry**
 - Solve real-world and mathematical problems involving area, surface area, and volume.
- **Statistics and Probability**
 - Develop understanding of statistical variability.
 - Summarize and describe distributions.

Emphases are given at the cluster level. Refer to the New Jersey Student Learning Standards for Mathematics for the specific standards that fall within each cluster. Even though the clusters have different levels of emphasis, this does not mean that standards can be omitted. All standards must be included in instruction throughout a grade level to ensure that students will be prepared for instruction at higher grade levels.

Indicator	Type	Cluster Heading
6.RP.A	M	Understand ratio concepts and use ratio reasoning to solve problems
6.NS.A	M	Analyze patterns and relationships

6.NS.B	A	Understand the place value system
6.NS.C	M	Perform operations with multi-digit whole numbers and with decimals to hundredths
6.EE.A	M	Use equivalent fractions as a strategy to add and subtract fractions
6.EE.B	M	Apply and extend previous understandings of multiplication and division to multiply and divide fractions
6.EE.C	M	Convert like measurement units within a given measurement system
6.G.A	S	Solve real-world and mathematical problems involving area, surface area, and volume
6.SP.A	A	Develop understanding of statistical variability
6.SP.B	A	Summarize and describe distributions

Key: M — Major, S — Supporting Clusters, A — Additional Clusters

Required Fluencies in Grade 6:

- **6.NS.B.2** Divide multi-digit whole numbers using the standard algorithm.
- **6.NS.B.3** Add, subtract, multiply and divide multi-digit decimals using the standard algorithm for each operation.

(NJDOE, 2024)

Grade 6 Units and Pacing	
Unit 1: Area and Surface Area	4.5 weeks
Unit 2: Ratios	4 weeks
Unit 3: Unit Rates and Percentages	4 weeks
Unit 4: Dividing Fractions	4 weeks
Unit 5: Arithmetic in Base Ten	3.5 weeks
Unit 6: Expressions and Equations	5 weeks
Unit 7: Rational Numbers	4 weeks
Unit 8: Data Sets and Distributions	4 weeks
Unit 9: Integrating Math Skills	2 weeks
Total Number of Weeks:	35 weeks

Unit 1: Area and Surface Area

Unit Goal: In this unit, students learn to find areas of polygons by decomposing, rearranging, and composing shapes. They learn to understand and use the terms “base” and “height,” and find areas of parallelograms and triangles. Students approximate areas of non-polygonal regions by polygonal regions. They represent polyhedra with nets and find their surface areas.
Duration: 5 weeks
Essential Questions: • How can we decompose and rearrange shapes to find the area of polygons effectively? • What role do the terms "base" and "height" play in finding the area of parallelograms and triangles?

- How can we approximate the area of non-polygonal regions using polygons?
- What strategies can we use to represent polyhedra with nets, and how does this help us find their surface areas?

Enduring Understandings:

- Decomposing and rearranging shapes allows us to simplify complex polygons into smaller, more manageable components, facilitating the calculation of their areas.
- Understanding the concept of "base" and "height" is essential for determining the area of parallelograms and triangles, as it provides a systematic approach to calculating their areas.
- Approximating the area of non-polygonal regions by using polygons helps us estimate areas in real-world scenarios and provides a practical application of polygonal area concepts.
- Representing polyhedra with nets enables us to visualize their surfaces and understand their three-dimensional structure, leading to the calculation of their surface areas through net manipulation.

Key NJSLS Strands:

- **6.G.A** Solve real-world and mathematical problems involving area, surface area, and volume.
 - **6.G.A.1** Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
 - **6.G.A.2** Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas and to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.
 - **6.G.A.4** Represent three-dimensional figures (e.g., pyramid, triangular prism, rectangular prism) using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.
- **6.EE.A** Apply and extend previous understandings of arithmetic to algebraic expressions
 - **6.EE.A.1** Write and evaluate numerical expressions involving whole-number exponents.
 - **6.EE.A.2** Write, read, and evaluate expressions in which letters stand for numbers.
 - **6.EE.A.2.a** Write expressions that record operations with numbers and with letters standing for numbers.
 - **6.EE.2c** Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole

number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). For example, use the formulas and to find the volume and surface area of a cube with sides of length .

Interdisciplinary Connections

- **NJSLSA.R7** Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words.
- **NJSLSA.SL1** Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively.
- **NJSLSA.SL4** Present information, findings, and supporting evidence such that listeners can follow the line of reasoning and the organization, development, and style are appropriate to task, purpose, and audience.
- **MS-ETS1-1** Students estimate sums and differences while defining the criteria and constraints of a design problem with sufficient precision to ensure a successful solution, taking into account relevant scientific principles

21st-Century Skills

- **9.1.8.FP.2** Evaluate the role of emotions, attitudes, and behavior (rational and irrational) in making financial decisions.

Career Ready Practices

- **9.2.8.CAP.3** Explain how career choices, educational choices, skills, economic conditions, and personal behavior affect income.

Technology Integration

- **9.4.8.TL.3** Select appropriate tools to organize and present information digitally.
- **9.4.8.TL.6** Collaborate to develop and publish work that provides perspectives on a real-world problem.

Skills	Instructional Topics	Assessment	Resources	Vocabulary
Find the area of right triangles and other triangles by composing into rectangles.	Tiling the Plane	Formative Assessments:	Materials:	area
Find the area of special quadrilaterals and polygons by composing into rectangles or decomposing into triangles and other shapes.	Finding Area by Decomposing and Rearranging	Diagnostic Assessment	1/2-inch cubes	region
Apply the techniques of finding area of polygons by composition or decomposition to solve real-world and mathematical problems.	Reasoning to Find Area	Mid-topic Checkpoint	1/2-inch graph paper	plane
Represent three-dimensional figures made up of rectangles and triangles by using nets.	Parallelograms	Lesson Quizzes	base-ten blocks	gap
Use the net to find the surface area of three-dimensional figures made up of rectangles and triangles	Bases and Heights of Parallelograms	Lesson Self-assessment	beakers	compose
	Area of Parallelograms	Teacher Observations	bingo chips	decompose
	Triangles	Class Discussions	blank paper	rearrange
	From Parallelograms to Triangles	Classwork/ Homework	colored pencils	two-dimensional
	Area of Triangles	Exit Slips/Google Form	cuisenaire rods	shaded
	Formula for the Area of a Triangle	Summative Assessments:	demonstration nets with and without flaps	strategy
	Bases and Heights of Triangles	Topic/Unit Assessments	dot stickers	parallelogram
	Polygons	Constructed Response Questions	empty containers	opposite (sides or angles)
	Surface Area	Projects	food coloring	quadrilateral
	What is Surface Area?	Cumulative Tests	four-function calculators	base (of a parallelogram or triangle)
		Performance Assessments	gallon-sized jug	height
			tracing paper	

Solve real-world and mathematical problems by using nets to find surface area applying net surface area techniques. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes. Show that volume of a right rectangular prism is the same when multiplying edge lengths or packing it with unit cubes. Find volumes of right rectangular prisms with fractional edge lengths applying the volume formulas $V = l w h$ and $V = B h$ in real-world or mathematical problems.	Polyhedra Nets and Surface Area More Nets, More Surface Area Distinguishing Between Surface Area and Volume Squares and Cubes Surface Area of a Cube	Alternate and Benchmark Assessments: Progress Monitoring Assessments Cumulative/Benchmark Assessments	graph paper colored pencils scissors index cards ruler protractor glue or glue sticks graduated cylinders hand-held graphing calculator graph paper grocery store circulars household items inch cubes index cards internet-enabled device liter-sized bottle markers masking tape	corresponding expression represent horizontal vertical identical parallelogram base (of a parallelogram or triangle) height compose decompose rearrange opposite vertex vertex edge polygon horizontal
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			measuring tapes	vertical
			metal paper fasteners (brass brads)	face
			meter sticks	surface area
			nets of polyhedra	area
			origami paper	region
			paper clips	polyhedron
			paper cups	net
			pattern blocks	prism
			pre-assembled or commercially produced polyhedra	pyramid
			pre-assembled or commercially produced tangrams	base (of a prism or pyramid)
			quart-sized bottle	three-dimensional
			rulers	polygon
			rulers marked with centimeters	vertex
			salt	edge
			scale	face
				prism
				pyramid

			a digital scale that can output in grams, kilograms, ounces, or pounds scissors snap cubes sticky notes stopwatches straightedges string tape teaspoon tools for creating a visual display tracing paper tray water yardsticks Activities: Patterned Tile Designs Area Puzzle Challenge	volume appropriate quantity two-dimensional three-dimensional squared cubed exponent edge length value (of an expression) squared cubed net estimate description surface area volume
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			Area Debate Parallelogram Properties Parallelogram Discovery Lab Parallelogram Area Challenge Triangle Investigation Parallelogram to Triangle Conversion Triangle Area Race Triangle Formula Practice Triangle Construction Challenge Polygon Area Gallery Walk Surface Area Exploration Stations Surface Area Scavenger Hunt	
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			Polyhedron Models Net Construction Challenge Polyhedron Net Investigation Surface Area vs. Volume Sort Square and Cube Exploration Cube Surface Area Challenge Designing a Tent Mentor Texts: <i>Flatland: A Romance of Many Dimensions</i> by Edwin A. Abbott <i>The Number Devil: A Mathematical Adventure</i> by Hans Magnus Enzensberger <i>The Adventures of Penrose the Mathematical Cat</i> by Theoni Pappas	
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Modifications and Accommodations	
Tier II and III Strategies	Small group instruction (Tier II) • Explicit instruction • Schema-based instruction • Use visuals • Use manipulatives • Use a multi-sensory approach • Guided notes and/or scaffold outline for written assignments • Introduce key vocabulary before lesson • Lesson taught again using a differentiated approach Small group or Individual instruction (Tier III) • Intensive instruction • Practice fact retrieval • Lesson taught again using a differentiated approach • Allow answers to be given orally or dictated • Use visuals / Anchor Charts
Students with IEPs and 504 Plans	<u>Universal Design for Learning and Access for Students with Disabilities</u> <u>Movematics (dyscalculia.org)</u> Follow all the IEP/504 plan modifications Allow extra time to complete assignments or tests Guided notes and/or scaffold outline for written assignments Work in a small group Solidify and refine concepts through repetition

	Allow answers to be given orally or dictated Use multi-sensory teaching approaches Utilize assistive technology and materials Use large print books, Braille, or books on CD (digital text) Text to speech/audio recorded selections Amplification system as needed Leveled texts according to ability Fine motor skill stations embedded in rotation as needed Modified or constrained spelling word lists Provide anchor charts with high frequency words and phonemic patterns Students might need guidance as to what actions happen during partner work time. Invite students to discuss what it looks like when they are working with their partner while exploring the connecting cubes. Generate a list of shared expectations for partner work. Invite students to plan a strategy before they begin working. If time allows, invite students to share their plan with a partner. Some students may benefit from feedback that emphasizes effort and time on task. For example, moving counters in the squares of the 5-frame or filling the 5-frame with one counter in each square.
Multi-language Learners	<u>Mathematical Language Development and Access for English Learners</u> Work toward longer passages as skills in English increase

	Use visuals Introduce/highlight key vocabulary before lesson Teacher models reading aloud daily Provide peer tutoring Utilize translation supports including bilingual dictionary, translation device, assistive technology, and peers Guided notes and/or scaffold outline for written assignments Provide students with English Learner leveled readers Support recommended by the grade specific “can do” list outlined by WIDA, including Key Uses Edition and Descriptor Name Charts Create a visual display of what students want to do or make with the connecting cubes. As students share ideas, include a drawing or annotation to illustrate connections. Invite students to use gestures as they count.
Accelerating Students	<u>Accelerating Students Guide</u> Create an enhanced set of introductory activities (e.g. advance organizers, concept maps, concept puzzles) Provide options, alternatives and choices to differentiate and broaden the curriculum Organize and offer flexible small group learning activities Provide whole group enrichment explorations

	Teach cognitive and methodological skills Use center, stations, or contracts Organize integrated problem-solving simulations Propose interest-based extension activities Expose students to beyond level texts.
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(DOE, 2024)

Unit 2: Ratios

Unit Goal: In this unit, students learn to understand and use the terms “ratio,” “rate,” “equivalent ratios,” “per,” “at this rate,” “constant speed,” and “constant rate,” and to recognize when two ratios are or are not equivalent. They represent ratios as expressions, and represent equivalent ratios with double number line diagrams, tape diagrams, and tables. They use these terms and representations in reasoning about situations involving color mixtures, recipes, unit pricing, and constant speed.

Duration: 4 weeks

Essential Questions:

- What are ratios and rates?
- How can you use ratios and rates to describe quantities and solve problems?
- What is the meaning of percent?
- How can percent be estimated and found

Enduring Understandings:

- Ratios can be used to describe the relationship between two quantities where every x units of one quantity, there are y units of another quantity.
- Equivalent ratios can be found by multiplying or dividing both terms by the same nonzero number.
- Equivalent ratios can be represented in a ratio table and the pairs of values can be plotted on a coordinate plane.
- A rate is a special type of ratio that compares two quantities with different measures whereas a unit rate is a special rate that

compares a quantity to one unit of another quantity.

- Rates are easily compared when they are expressed as unit rates.
- Unit rates, including unit prices, can be used to solve problems.
- Unit rates and conversion factors can be used to convert customary and metric units of measure.
- A percent is a rate in which the first term is compared to 100. The percent is the number of hundredths that represent the part of the whole.
- Fractions, decimals, and percents are three ways to show parts of a whole.
- A percent greater than 100 is equivalent to more than the whole. A percent less than 1 is equivalent to less than 1/100 of the whole.
- Equivalent fractions and compatible numbers can be used to estimate the percent of a number.
- Finding the percent of a whole is like finding the fractional part of a whole.
- Models and equations can be used to find the whole amount when the percent and a part are known.

Key NJSLS Strands:

- **6.RP.A** Understand ratio concepts and use ratio reasoning to solve problems
 - **6.RP.A.1** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”
 - **6.RP.A.2** Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is $\frac{3}{4}$ -cup of flour for each cup of sugar.” “We paid \$75 for 15 hamburgers, which is a rate of \$5 per hamburger.” (Clarification: Expectations for unit rates in this grade are limited to non-complex fractions.)
 - **6.RP.A.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
 - **6.RP.A.3a** Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
 - **6.RP.A.3.b** Solve unit rate problems including those involving unit pricing and constant speed.

Interdisciplinary Connections				
● NJSLSA.R7 Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words. ● NJSLSA.SL1 Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively. ● NJSLSA.SL4 Present information, findings, and supporting evidence such that listeners can follow the line of reasoning and the organization, development, and style are appropriate to task, purpose, and audience. ● MS-PS3-3 Apply scientific principles to design, construct, and test a device that either minimizes or maximizes thermal energy transfer.				
21st-Century Skills				
● 9.2.8.CAP.3 Explain how career choices, educational choices, skills, economic conditions, and personal behavior affect income.				
Career Ready Practices				
● CRP2 Apply appropriate academic and technical skills. ● CRP8 Utilize critical thinking to make sense of problems and persevere in solving them. ● CRP11 Use technology to enhance productivity.				
Technology Integration				
● 9.4.8.TL.3 Select appropriate tools to organize and present information digitally. ● 9.4.8.TL.6 Collaborate to develop and publish work that provides perspectives on a real-world problem.				

Skills	Instructional Topics	Assessment	Resources	Vocabulary
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Explain the concept of a ratio through definition. Use ratio language to describe a relationship between two quantities. Represent equivalent ratios with double number line diagrams, tape diagrams, and tables. Recognize when two ratios are or are not equivalent. Represent ratios as expressions. Express a ratio relationship using rate language Represent and solve rate and ratio real-world and mathematical problems by using tables, tape diagrams, double number line diagrams, and equations Create tables of equivalent ratios and	What are Ratios? Introducing Ratios and Ratio Language Representing Ratios with Diagrams Equivalent Ratios Recipes Color Mixtures Defining Equivalent Ratios Representing Equivalent Ratios Introducing Double Number Line Diagrams Creating Double Number Line Diagrams How Much for One? Constant Speed Comparing Situations by Examining Ratios Solving Ratio and Rate Problems	Formative Assessments: Diagnostic Assessment Mid-topic Checkpoint Lesson Quizzes Lesson Self-assessment Teacher Observations Class Discussions Classwork/ Homework Exit Slips/Google Form Summative Assessments: Topic/Unit Assessments Constructed Response Questions Projects Cumulative Tests Performance Assessments	Materials: 1/2-inch cubes 1/2-inch graph paper base-ten blocks beakers bingo chips blank paper colored pencils cuisenaire rods decks of playing cards demonstration nets with and without flaps dot stickers empty containers food coloring four-function calculators gallon-sized jug tracing paper graph paper	rate ratio ___ to ___ ___ for every ___ diagram recipe batch same taste mixture same color check (an answer) equivalent ratios double number line diagram tick marks representation per unit price
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find missing values with whole number measurements Plot pairs of values, in the coordinate plane, from a ratio table to compare ratios	Representing Ratios with Tables Navigating a Table of Equivalent Ratios Tables and Double Number Line Diagrams Solving Equivalent Ratio Problems Part-part-whole Ratios	Alternate and Benchmark Assessments: Progress Monitoring Assessments Cumulative/Benchmark Assessments	colored pencils scissors index cards ruler protractor glue or glue sticks graduated cylinders hand-held graphing calculator graph paper grocery store circulars household items inch cubes index cards internet-enabled device liter-sized bottle markers masking tape measuring tapes	how much for 1 at this rate meters per second constant speed same rate constant rate equivalent ratios table row column calculation per tape diagram parts suppose
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			metal paper fasteners (brass brads) meter sticks nets of polyhedra origami paper paper clips paper cups pattern blocks pre-assembled or commercially produced polyhedra pre-assembled or commercially produced tangrams quart-sized bottle rulers rulers marked with centimeters salt scale a digital scale that can output in grams,	
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			kilograms, ounces, or pounds scissors snap cubes sticky notes stopwatches straightedges string tape teaspoon tools for creating a visual display tracing paper tray water yardsticks Activities: What are Ratios?: Ratio Exploration	
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			Introducing Ratios and Ratio Language: Ratio Vocabulary Match Ratio Diagram Challenge Equivalent Ratio Puzzles Recipes: Cooking with Ratios Color Mixing Lab Defining Equivalent Ratios Equivalent Ratio Definitions Equivalent Ratio Models Double Number Line Introduction Creating Double Number Line Diagrams Double Number Line Creations How Much for One?	
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			Unit Rate Exploration Constant Speed Speed Challenge Ratio Comparison Tasks Problem-Solving Scenarios Ratio Table Construction Navigating a Table of Equivalent Ratios Equivalent Ratio Table Challenge Tables and Double Number Line Diagrams Modeling Ratios Solving Equivalent Ratio Problems Equivalent Ratio Problem Solving Part-Part-Whole Exploration	
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			Advanced Ratio Problem Solving Mentor Texts: <i>Percy Jackson and the Olympians</i> series by Rick Riordan <i>Sir Cumference and All the King's Tens</i> by Cindy Neuschwander	
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Modifications and Accommodations	
Tier II and III Strategies	Small group instruction (Tier II) ● Explicit instruction ● Schema-based instruction ● Use visuals ● Use manipulatives ● Use a multi-sensory approach ● Guided notes and/or scaffold outline for written assignments ● Introduce key vocabulary before lesson ● Lesson taught again using a differentiated approach Small group or Individual instruction (Tier III) ● Intensive instruction ● Practice fact retrieval ● Lesson taught again using a differentiated approach ● Allow answers to be given orally or dictated ● Use visuals / Anchor Charts
Students with IEPS and 504 Plans	<u>Universal Design for Learning and Access for Students with Disabilities</u>

[Movematics \(dyscalculia.org\)](http://dyscalculia.org)

Follow all the IEP/504 plan modifications

Allow extra time to complete assignments or tests

Guided notes and/or scaffold outline for written assignments

Work in a small group

Solidify and refine concepts through repetition

Allow answers to be given orally or dictated

Use multi-sensory teaching approaches

Utilize assistive technology and materials

Use large print books, Braille, or books on CD (digital text)

Text to speech/audio recorded selections

Amplification system as needed

Leveled texts according to ability

Fine motor skill stations embedded in rotation as needed

Modified or constrained spelling word lists

Provide anchor charts with high frequency words and phonemic patterns

Students might need guidance as to what actions happen during partner work time. Invite students to discuss what it looks like when they are working with their partner while exploring the connecting cubes. Generate a list of shared expectations for partner work.

Invite students to plan a strategy before they begin working. If time allows, invite

	students to share their plan with a partner. Some students may benefit from feedback that emphasizes effort and time on task. For example, moving counters in the squares of the 5-frame or filling the 5-frame with one counter in each square.
Multi-language Learners	<u>Mathematical Language Development and Access for English Learners</u> Work toward longer passages as skills in English increase Use visuals Introduce/highlight key vocabulary before lesson Teacher models reading aloud daily Provide peer tutoring Utilize translation supports including bilingual dictionary, translation device, assistive technology, and peers Guided notes and/or scaffold outline for written assignments Provide students with English Learner leveled readers Support recommended by the grade specific “can do” list outlined by WIDA, including Key Uses Edition and Descriptor Name Charts Create a visual display of what students want to do or make with the connecting cubes. As students share ideas, include a drawing or annotation to illustrate connections. Invite students to use gestures as they count.
Accelerating Students	<u>Accelerating Students Guide</u>

	Create an enhanced set of introductory activities (e.g. advance organizers, concept maps, concept puzzles) Provide options, alternatives and choices to differentiate and broaden the curriculum Organize and offer flexible small group learning activities Provide whole group enrichment explorations Teach cognitive and methodological skills Use center, stations, or contracts Organize integrated problem-solving simulations Propose interest-based extension activities Expose students to beyond level texts.
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(DOE, 2024)

Unit 3: Unit Rates and Percentages

Unit Goal: Students learn to understand and use the terms “unit rate,” “speed,” “pace,” “percent,” and “percentage,” and recognize that equivalent ratios have equal unit rates. They represent percentages with tables, tape diagrams, and double number line diagrams, and as expressions. They use these terms and representations in reasoning about situations involving unit price, constant speed, and measurement conversion.
Duration: 4 weeks
Essential Questions: • How do unit rate, speed, pace, percent, and percentage relate to each other, and how are they used in different contexts? • Why is it important to recognize that equivalent ratios have equal unit rates, and how does this understanding help in problem-solving?

- How can percentages be represented using tables, tape diagrams, double number line diagrams, and expressions, and how do these representations aid in understanding and problem-solving?
- In what ways can unit price, constant speed, and measurement conversion be reasoned about using unit rate, speed, pace, percent, and percentage?

Enduring Understandings:

- Understanding the terms unit rate, speed, pace, percent, and percentage provides a foundation for analyzing and comparing quantities in various contexts.
- Recognizing that equivalent ratios have equal unit rates facilitates problem-solving by enabling the comparison of different situations and quantities.
- Representing percentages with tables, tape diagrams, double number line diagrams, and expressions helps visualize and interpret percentages in real-world scenarios.
- Unit price, constant speed, and measurement conversion can be reasoned about using unit rate, speed, pace, percent, and percentage, allowing for effective problem-solving and decision-making in practical situations.

Key NJSLS Strands:

- **6.RP.A** Understand ratio concepts and use ratio reasoning to solve problems
 - **6.RP.A.2** Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is $\frac{3}{4}$ -cup of flour for each cup of sugar.” “We paid \$75 for 15 hamburgers, which is a rate of \$5 per hamburger.” (Clarification: Expectations for unit rates in this grade are limited to non-complex fractions.)
 - **6.RP.A.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
 - **6.RP.A.3.b** Solve unit rate problems including those involving unit pricing and constant speed.
 - **6.RP.A.3.c** Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means times the quantity); solve problems involving finding the whole, given a part and the percent.
 - **6.RP.A.3.d** Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.
- **6.G.A** Solve real-world and mathematical problems involving area, surface area, and volume.

Interdisciplinary Connections
● NJSLSA.R7 Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words. ● NJSLSA.SL1 Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively. ● NJSLSA.SL4 Present information, findings, and supporting evidence such that listeners can follow the line of reasoning and the organization, development, and style are appropriate to task, purpose, and audience.
21st-Century Skills
● 9.1.8.PB.2 Explain how different circumstances can affect one's personal budget. ● 9.1.8.PB.4 Construct a simple personal savings and spending plan based on various sources of income and different stages of life (e.g. teenager, young adult, family). ● 9.1.8.PB.6 Construct a budget to save for short-term, long-term, and charitable goals.
Career Ready Practices
● CRP2 Apply appropriate academic and technical skills. ● CRP8 Utilize critical thinking to make sense of problems and persevere in solving them. ● CRP11 Use technology to enhance productivity.
Technology Integration
● 9.4.5.CT.3 Describe how digital tools and technology may be used to solve problems.

Skills	Instructional Topics	Assessment	Resources	Vocabulary
Construct a unit rate (a/b) from a given ratio	Units of Measurement	Formative Assessments:	Materials: 1/2-inch cubes	unit rate

(a:b) Explain a unit rate (a/b) associated with a ratio (a:b) Find the two values a/b and b/a that are associated with the ratio $a:b$, and interpret them as rates per 1. Compare two values (a/b or b/a) and determine which is more useful than the other in reasoning about a given situation. Solve unit rate problems, including unit pricing and constant speed Explain if two ratios a/b and c/d are equivalent, then $a/b=c/d$. The values a/b and c/d are called <i>unit rates</i> Internalize the meaning of important benchmark percentages, for	Unit Conversion Anchoring Units of Measurement Measuring with Different-Sized Units Converting Units Rates Comparing Speeds and Prices Interpreting Rates Equivalent Ratios Have the Same Unit Rates More about Constant Speed Solving Rate Problems Percentages What Are Percentages? Percentages and Double Number Lines Percentages and Tape Diagrams	Diagnostic Assessment Mid-topic Checkpoint Lesson Quizzes Lesson Self-assessment Teacher Observations Class Discussions Classwork/ Homework Exit Slips/Google Form Summative Assessments: Topic/Unit Assessments Constructed Response Questions Projects Cumulative Tests Performance Assessments Alternate and Benchmark Assessments:	1/2-inch graph paper base-ten blocks beakers bingo chips blank paper colored pencils cuisenaire rods decks of playing cards demonstration nets with and without flaps dot stickers empty containers food coloring four-function calculators gallon-sized jug tracing paper graph paper colored pencils scissors	speed pace percent percentage
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example, connect “75% of a number” with “$\frac{3}{4}$ times a number” and “0.75 times a number.” Find the part, whole, and percent of a quantity in real-world problems Convert measurement units utilizing ratio reasoning	Benchmark Percentages Solving Percentage Problems Finding This Percent of That Finding the Percentage Painting a Room	Progress Monitoring Assessments Cumulative/Benchmark Assessments	index cards ruler protractor glue or glue sticks graduated cylinders hand-held graphing calculator graph paper grocery store circulars household items inch cubes index cards internet-enabled device liter-sized bottle markers masking tape measuring tapes metal paper fasteners (brass brads) meter sticks	
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			nets of polyhedra origami paper paper clips paper cups pattern blocks pre-assembled or commercially produced polyhedra pre-assembled or commercially produced tangrams quart-sized bottle rulers rulers marked with centimeters salt scale a digital scale that can output in grams, kilograms, ounces, or pounds scissors	
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			snap cubes sticky notes stopwatches straightedges swtring tape teaspoon tools for creating a visual display tracing paper tray water yardsticks Activities: Measurement Scavenger Hunt Conversion Challenge Anchor Measurement Tasks Measurement Relay Race	
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			Unit Conversion Puzzles Rate Investigation Speed vs. Price Challenge Rate Analysis Task Equivalent Ratio Match-Up Constant Speed Simulation Rate Word Problem Solving Percentage Introduction Task Percentage Scenarios Double Number Line Percentage Representation Tape Diagram Percentage Modeling Benchmark Percentage Exploration	
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			Percentage Problem Solving Tasks Percentage Task Cards Percentage Mystery Room Painting Project Mentor Texts: <i>The Lemonade War</i> by Jacqueline Davies	
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Modifications and Accommodations	
Tier II and III Strategies	Small group instruction (Tier II) ● Explicit instruction ● Schema-based instruction ● Use visuals ● Use manipulatives ● Use a multi-sensory approach ● Guided notes and/or scaffold outline for written assignments ● Introduce key vocabulary before lesson ● Lesson taught again using a differentiated approach Small group or Individual instruction (Tier III) ● Intensive instruction ● Practice fact retrieval ● Lesson taught again using a differentiated approach ● Allow answers to be given orally or dictated

	● Use visuals / Anchor Charts
Students with IEPs and 504 Plans	<u>Universal Design for Learning and Access for Students with Disabilities</u> <u>Movematics (dyscalculia.org)</u> Follow all the IEP/504 plan modifications Allow extra time to complete assignments or tests Guided notes and/or scaffold outline for written assignments Work in a small group Solidify and refine concepts through repetition Allow answers to be given orally or dictated Use multi-sensory teaching approaches Utilize assistive technology and materials Use large print books, Braille, or books on CD (digital text) Text to speech/audio recorded selections Amplification system as needed Leveled texts according to ability Fine motor skill stations embedded in rotation as needed Modified or constrained spelling word lists Provide anchor charts with high frequency words and phonemic patterns

	Students might need guidance as to what actions happen during partner work time. Invite students to discuss what it looks like when they are working with their partner while exploring the connecting cubes. Generate a list of shared expectations for partner work. Invite students to plan a strategy before they begin working. If time allows, invite students to share their plan with a partner. Some students may benefit from feedback that emphasizes effort and time on task. For example, moving counters in the squares of the 5-frame or filling the 5-frame with one counter in each square.
Multi-language Learners	<u>Mathematical Language Development and Access for English Learners</u> Work toward longer passages as skills in English increase Use visuals Introduce/highlight key vocabulary before lesson Teacher models reading aloud daily Provide peer tutoring Utilize translation supports including bilingual dictionary, translation device, assistive technology, and peers Guided notes and/or scaffold outline for written assignments Provide students with English Learner leveled readers Support recommended by the grade specific “can do” list outlined by WIDA, including Key Uses Edition and Descriptor Name Charts Create a visual display of what students want to do or make with the connecting cubes.

	As students share ideas, include a drawing or annotation to illustrate connections. Invite students to use gestures as they count.
Accelerating Students	<u>Accelerating Students Guide</u> Create an enhanced set of introductory activities (e.g. advance organizers, concept maps, concept puzzles) Provide options, alternatives and choices to differentiate and broaden the curriculum Organize and offer flexible small group learning activities Provide whole group enrichment explorations Teach cognitive and methodological skills Use center, stations, or contracts Organize integrated problem-solving simulations Propose interest-based extension activities Expose students to beyond level texts.

(DOE, 2024)

Unit 4: Dividing Fractions

Unit Goal: In this unit, students examine how the relative sizes of numerator and denominator affect the size of their quotient when numerator or denominator (or both) is a fraction. They acquire the understanding that dividing by a/b has the same outcome as multiplying by b , then by $1/a$. They compute quotients of fractions. They solve problems involving lengths and areas of figures with fractional side lengths and extend the formula for the volume of a right rectangular prism to prisms with fractional edge lengths and use it to solve problems. They use tape diagrams, equations, and expressions to represent situations involving partitive or

quotative interpretations of division with fractions. Given a multiplication or division equation or expression with fractions, they describe a situation that it could represent. They use tape diagrams and equations in reasoning about situations that involve multiplication and division of fractions.

Duration: 4 weeks

Essential Questions:

- How does the relative sizes of numerator and denominator affect the size of the quotient when dividing fractions?
- What is the relationship between dividing by a fraction and multiplying by its reciprocal?
- How can we compute quotients of fractions efficiently and accurately?
- In what ways can fractions be applied to solve problems involving lengths, areas, and volumes?
- How do tape diagrams, equations, and expressions help represent situations involving division with fractions?
- How can multiplication and division of fractions be reasoned about using tape diagrams and equations?

Enduring Understandings:

- The relative sizes of the numerator and denominator in a fraction determine the size of the quotient when dividing fractions, with smaller denominators resulting in larger quotients.
- Dividing by a fraction is equivalent to multiplying by its reciprocal, providing a practical shortcut for division involving fractions.
- Proficiency in computing quotients of fractions enables efficient problem-solving in various mathematical contexts.
- Fractions can be effectively utilized to solve problems related to lengths, areas, and volumes, extending mathematical concepts beyond whole numbers.
- Tape diagrams, equations, and expressions serve as valuable tools for representing situations involving division with fractions, facilitating problem-solving and reasoning.
- Reasoning about multiplication and division of fractions using tape diagrams and equations enhances conceptual understanding and problem-solving skills in mathematical contexts.

Key NJSL Strands:

- **6.NS.A** Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
 - **6.NS.A.1** Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.

- **6.G.A** Solve real-world and mathematical problems involving area, surface area, and volume.
 - **6.G.A.1** Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
 - **6.G.A.2** Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas and to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

Interdisciplinary Connections

- **NJSLSA.R7** Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words.
- **NJSLSA.SL1** Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively.
- **NJSLSA.SL4** Present information, findings, and supporting evidence such that listeners can follow the line of reasoning and the organization, development, and style are appropriate to task, purpose, and audience.
- **MS-ETS1-1** Students estimate sums and differences while defining the criteria and constraints of a design problem with sufficient precision to ensure a successful solution, taking into account relevant scientific principles.

21st-Century Skills

- **9.4.8.CT.2** Develop multiple solutions to a problem and evaluate short- and long-term effects to determine the most plausible option (e.g., MS-ETS1-4, 6.1.8.CivicsDP.1).
- **9.4.8.IML.12** Use relevant tools to produce, publish, and deliver information supported with evidence for an authentic audience.

Career Ready Practices

- **CRP2** Apply appropriate academic and technical skills.

Technology Integration

- **9.4.5.CT.3** Describe how digital tools and technology may be used to solve problems.

Skills	Instructional Topics	Assessment	Resources	Vocabulary
Examine how the relative sizes of numerator and denominator affect the size of their quotient when numerator or denominator (or both) is a fraction. Understand that dividing by a/b has the same outcome as multiplying by a/b, then b by $1/a$. Compute quotients of fractions. Interpret quotients of fractions. Solve word problems involving division of fractions by fractions using visual models and	Making Sense of Division Size of Divisor and Size of Quotient Meanings of Division Interpreting Division Situations Meanings of Fraction Division How Many Groups? Using Diagrams to Find the Number of Groups What Fraction of a Group? How Much in Each Group? Algorithm for Fraction	Formative Assessments: Diagnostic Assessment Mid-topic Checkpoint Lesson Quizzes Lesson Self-assessment Teacher Observations Class Discussions Classwork/ Homework Exit Slips/Google Form Summative Assessments: Topic/Unit Assessments Constructed Response Questions	Materials: 1/2-inch cubes 1/2-inch graph paper base-ten blocks beakers bingo chips blank paper colored pencils cuisenaire rods decks of playing cards demonstration nets with and without flaps dot stickers empty containers	diagram recipe batch same taste mixture same color check (an answer) equivalent ratios double number line diagram tick marks representation per

equations. Solve problems involving lengths and areas of figures with fractional side lengths and extend the formula for the volume of a right rectangular prism to prisms with fractional edge lengths and use it to solve problems. Use tape diagrams, equations, and expressions to represent situations involving partitive or quotitive interpretations of division with fractions. Use tape diagrams and equations in reasoning about situations that involve multiplication and division of fractions. Given a multiplication or division equation or expression with fractions, describe a situation that it could	Division Dividing by Unit and Non-Unit Fractions Using an Algorithm to Divide Fractions Fractions in Lengths, Areas, and Volumes Fractional Lengths Rectangles with Fractional Side Lengths Fractional Lengths in Triangles and Prisms Volume of Prisms Solving Problems Involving Fractions Fitting Boxes into Boxes	Projects Cumulative Tests Performance Assessments Alternate and Benchmark Assessments: Progress Monitoring Assessments Cumulative/Benchmark Assessments	food coloring four-function calculators gallon-sized jug tracing paper graph paper colored pencils scissors index cards ruler protractor glue or glue sticks graduated cylinders hand-held graphing calculator graph paper grocery store circulars household items inch cubes index cards	unit price how much for 1 at this rate meters per second constant speed
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represent.			internet-enabled device liter-sized bottle markers masking tape measuring tapes metal paper fasteners (brass brads) meter sticks nets of polyhedra origami paper paper clips paper cups pattern blocks pre-assembled or commercially produced polyhedra pre-assembled or commercially produced tangrams quart-sized bottle rulers	
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			rulers marked with centimeters salt scale a digital scale that can output in grams, kilograms, ounces, or pounds scissors snap cubes sticky notes stopwatches straightedges string tape teaspoon tools for creating a visual display tracing paper tray water	
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			yardsticks Activities: Division Detective Divisor vs. Quotient Comparison Real-World Division Scenarios Word Problem Solving Fraction Division Exploration Grouping Tasks Diagrammatic Division Fractional Grouping Tasks Fair Sharing Activities Fraction Division Practice Unit and Non-Unit Fraction Division Fraction Division Algorithm Mastery	
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			Fractional Measurement Exploration Measuring with Fractional Units Area of Rectangles with Fractional Dimensions Volume of Prisms with Fractional Dimensions Prism Volume Problem Solving Fraction Problem-Solving Tasks Volume Packing Challenges Mentor Texts: <i>The Math Inspectors:</i> <i>The Case of the Claymore Diamond</i> by Daniel Kenney	
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Modifications and Accommodations	
Tier II and III Strategies	Small group instruction (Tier II) • Explicit instruction • Schema-based instruction • Use visuals

	● Use manipulatives ● Use a multi-sensory approach ● Guided notes and/or scaffold outline for written assignments ● Introduce key vocabulary before lesson ● Lesson taught again using a differentiated approach Small group or Individual instruction (Tier III) ● Intensive instruction ● Practice fact retrieval ● Lesson taught again using a differentiated approach ● Allow answers to be given orally or dictated ● Use visuals / Anchor Charts
Students with IEPS and 504 Plans	<u>Universal Design for Learning and Access for Students with Disabilities</u> <u>Movematics (dyscalculia.org)</u> Follow all the IEP/504 plan modifications Allow extra time to complete assignments or tests Guided notes and/or scaffold outline for written assignments Work in a small group Solidify and refine concepts through repetition Allow answers to be given orally or dictated Use multi-sensory teaching approaches Utilize assistive technology and materials Use large print books, Braille, or books on CD (digital text)

	Text to speech/audio recorded selections Amplification system as needed Leveled texts according to ability Fine motor skill stations embedded in rotation as needed Modified or constrained spelling word lists Provide anchor charts with high frequency words and phonemic patterns Students might need guidance as to what actions happen during partner work time. Invite students to discuss what it looks like when they are working with their partner while exploring the connecting cubes. Generate a list of shared expectations for partner work. Invite students to plan a strategy before they begin working. If time allows, invite students to share their plan with a partner. Some students may benefit from feedback that emphasizes effort and time on task. For example, moving counters in the squares of the 5-frame or filling the 5-frame with one counter in each square.
Multi-language Learners	<u>Mathematical Language Development and Access for English Learners</u> Work toward longer passages as skills in English increase Use visuals Introduce/highlight key vocabulary before lesson Teacher models reading aloud daily Provide peer tutoring Utilize translation supports including bilingual dictionary, translation device, assistive

	technology, and peers Guided notes and/or scaffold outline for written assignments Provide students with English Learner leveled readers Support recommended by the grade specific “can do” list outlined by WIDA, including Key Uses Edition and Descriptor Name Charts Create a visual display of what students want to do or make with the connecting cubes. As students share ideas, include a drawing or annotation to illustrate connections. Invite students to use gestures as they count.
Accelerating Students	<u>Accelerating Students Guide</u> Create an enhanced set of introductory activities (e.g. advance organizers, concept maps, concept puzzles) Provide options, alternatives and choices to differentiate and broaden the curriculum Organize and offer flexible small group learning activities Provide whole group enrichment explorations Teach cognitive and methodological skills Use center, stations, or contracts Organize integrated problem-solving simulations Propose interest-based extension activities Expose students to beyond level texts.

Unit 5: Base Ten - Decimals

Unit Goal: In this unit, students compute sums, differences, products, and quotients of multi-digit whole numbers and decimals, using efficient algorithms. They use calculations with whole numbers and decimals to solve problems set in real-world contexts.

Duration: 3.5 weeks

Essential Questions:

- How can efficient algorithms be used to compute sums, differences, products, and quotients of multi-digit whole numbers and decimals?
- What strategies can we employ to solve real-world problems involving calculations with whole numbers and decimals?
- How do we determine which operation (addition, subtraction, multiplication, or division) is appropriate for solving a given real-world problem?
- What are the similarities and differences between computing with whole numbers and decimals?
- How can we ensure the accuracy and efficiency of our calculations with whole numbers and decimals?
- In what ways can mathematical operations be applied to real-world contexts to solve practical problems and make informed decisions?

Enduring Understandings:

- Efficient algorithms can streamline the process of computing sums, differences, products, and quotients of multi-digit whole numbers and decimals, leading to accurate and efficient problem-solving.
- The ability to apply mathematical operations to real-world contexts empowers students to solve practical problems, make informed decisions, and understand the relevance of mathematics in everyday life.
- Mastery of computational skills with both whole numbers and decimals allows students to navigate a wide range of mathematical tasks and scenarios with confidence and precision.
- Problem-solving strategies, such as identifying key information, selecting appropriate operations, and interpreting results, are essential for applying mathematical concepts to real-world situations.
- Understanding the similarities and differences between calculations with whole numbers and decimals enhances students' ability to generalize concepts and apply them flexibly across different contexts.

Key NJSLS Strands:

- **6.NS.B** Compute fluently with multi-digit numbers & find common factors & multiple
 - **6.NS.B.2** With accuracy and efficiency, divide multi-digit numbers using the standard algorithm.
 - **6.NS.B.3** With accuracy and efficiency, add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.
 - **6.NS.B.4** Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor.
- **6.EE.A** Apply and extend previous understandings of arithmetic to algebraic expressions
 - **6.EE.A.1** Write and evaluate numerical expressions involving whole-number exponents.
 - **6.EE.A.2** Write, read, and evaluate expressions in which letters stand for numbers.
 - **6.EE.A.2.a** Write expressions that record operations with numbers and with letters standing for numbers.
 - **6.EE.A.2.b** Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity.
 - **6.EE.A.2.c** Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length s .
 - **6.EE.A.3** Apply the properties of operations to generate equivalent expressions.
 - **6.EE.A.4** Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them).

Interdisciplinary Connections

- **NJSLSA.R7** Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words.
- **NJSLSA.SL1** Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively.

● NJSLSA.SL4 Present information, findings, and supporting evidence such that listeners can follow the line of reasoning and the organization, development, and style are appropriate to task, purpose, and audience. ● MS-ETS1-1 Students estimate sums and differences while defining the criteria and constraints of a design problem with sufficient precision to ensure a successful solution, taking into account relevant scientific principles.
21st-Century Skills
● 9.2.8.CAP.3 Explain how career choices, educational choices, skills, economic conditions, and personal behavior affect income.
Career Ready Practices
● CRP4 Communicate clearly and effectively with reason.
Technology Integration
● 9.4.8.TL.6 Collaborate to develop and publish work that provides perspectives on a real-world problem.

Skills	Instructional Topics	Assessment	Resources	Vocabulary
Divide multi-digit numbers using the standard algorithm working towards accuracy and efficiency. Add, subtract, multiply, and divide multi-digit whole numbers and decimals using the standard algorithm for each operation, working	Warming Up to Decimals Using Decimals in a Shopping Context Adding and Subtracting Decimals Using Diagrams to Represent Addition and Subtraction	Formative Assessments: Diagnostic Assessment Mid-topic Checkpoint Lesson Quizzes Lesson Self-assessment Teacher Observations	Materials: 1/2-inch cubes 1/2-inch graph paper base-ten blocks beakers bingo chips blank paper	digits budget at least base-ten diagram bundle vertical calculation

towards accuracy and efficiency. Use calculations with whole numbers and decimals to solve problems set in real-world contexts. Find the greatest common factor of two whole numbers that are less than or equal to 100. Find the least common multiple of two whole numbers that are less than or equal to 12. Use the distributive property to factor the greatest common factor from a sum of two whole numbers in the range 1 to 100.	Adding and Subtracting Decimals with Few Non-Zero Digits Adding and Subtracting Decimals with Many Non-Zero Digits Multiplying Decimals Decimal Points in Products Methods for Multiplying Decimals Using Diagrams to Represent Multiplication Calculating Products of Decimals Dividing Decimals Using the Partial Quotients Method Using Long Division Dividing Numbers that Result in Decimals Dividing Decimals by Whole Numbers	Class Discussions Classwork/ Homework Exit Slips/Google Form Summative Assessments: Topic/Unit Assessments Constructed Response Questions Projects Cumulative Tests Performance Assessments Alternate and Benchmark Assessments: Progress Monitoring Assessments Cumulative/Benchmark Assessments	colored pencils cuisenaire rods decks of playing cards demonstration nets with and without flaps dot stickers empty containers food coloring four-function calculators gallon-sized jug tracing paper graph paper colored pencils scissors index cards ruler protractor glue or glue sticks graduated cylinders	place value digits unbundle method powers of 10 product decimal point partial products method partial quotients remainder long division divisor long division precision accuracy operation
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	Dividing Decimals by Decimals Let's Put It to Work Using Operations on Decimals to Solve Problems Making and Measuring Boxes		hand-held graphing calculator graph paper grocery store circulars household items inch cubes index cards internet-enabled device liter-sized bottle markers masking tape measuring tapes metal paper fasteners (brass brads) meter sticks nets of polyhedra origami paper paper clips paper cups pattern blocks	
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			pre-assembled or commercially produced polyhedra pre-assembled or commercially produced tangrams quart-sized bottle rulers rulers marked with centimeters salt scale a digital scale that can output in grams, kilograms, ounces, or pounds scissors snap cubes sticky notes stopwatches straightedges string	
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			tape teaspoon tools for creating a visual display tracing paper tray water yardsticks Activities: Decimal Dice Game Grocery Store Simulation Decimal Dominoes Decimal Bar Models Decimal Word Problems Decimal Challenge Decimal Arrays Decimal Place Value Chart Algorithm Exploration	
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			Decimal Area Models Decimal Product Puzzles Partial Quotients Practice Decimal Long Division Drills Real-World Division Problems Divide and Conquer Decimal Division Challenges Decimal Problem-Solving Tasks Box Building Project Mentor Texts: <i>Chasing Vermeer</i> by Blue Balliett <i>A Wrinkle in Time</i> by Madeleine L'Engle <i>The Secret of the Old Clock</i> by Carolyn Keen	
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Modifications and Accommodations	
Tier II and III Strategies	Small group instruction (Tier II) • Explicit instruction • Schema-based instruction • Use visuals • Use manipulatives • Use a multi-sensory approach • Guided notes and/or scaffold outline for written assignments • Introduce key vocabulary before lesson • Lesson taught again using a differentiated approach Small group or Individual instruction (Tier III) • Intensive instruction • Practice fact retrieval • Lesson taught again using a differentiated approach • Allow answers to be given orally or dictated • Use visuals / Anchor Charts
Students with IEPs and 504 Plans	<u>Universal Design for Learning and Access for Students with Disabilities</u> <u>Movematics (dyscalculia.org)</u> Follow all the IEP/504 plan modifications Allow extra time to complete assignments or tests Guided notes and/or scaffold outline for written assignments Work in a small group Solidify and refine concepts through repetition

	Allow answers to be given orally or dictated Use multi-sensory teaching approaches Utilize assistive technology and materials Use large print books, Braille, or books on CD (digital text) Text to speech/audio recorded selections Amplification system as needed Leveled texts according to ability Fine motor skill stations embedded in rotation as needed Modified or constrained spelling word lists Provide anchor charts with high frequency words and phonemic patterns Students might need guidance as to what actions happen during partner work time. Invite students to discuss what it looks like when they are working with their partner while exploring the connecting cubes. Generate a list of shared expectations for partner work. Invite students to plan a strategy before they begin working. If time allows, invite students to share their plan with a partner. Some students may benefit from feedback that emphasizes effort and time on task. For example, moving counters in the squares of the 5-frame or filling the 5-frame with one counter in each square.
Multi-language Learners	<u>Mathematical Language Development and Access for English Learners</u> Work toward longer passages as skills in English increase

	Use visuals Introduce/highlight key vocabulary before lesson Teacher models reading aloud daily Provide peer tutoring Utilize translation supports including bilingual dictionary, translation device, assistive technology, and peers Guided notes and/or scaffold outline for written assignments Provide students with English Learner leveled readers Support recommended by the grade specific “can do” list outlined by WIDA, including Key Uses Edition and Descriptor Name Charts Create a visual display of what students want to do or make with the connecting cubes. As students share ideas, include a drawing or annotation to illustrate connections. Invite students to use gestures as they count.
Accelerating Students	<u>Accelerating Students Guide</u> Create an enhanced set of introductory activities (e.g. advance organizers, concept maps, concept puzzles) Provide options, alternatives and choices to differentiate and broaden the curriculum Organize and offer flexible small group learning activities Provide whole group enrichment explorations

	Teach cognitive and methodological skills Use center, stations, or contracts Organize integrated problem-solving simulations Propose interest-based extension activities Expose students to beyond level texts.
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(DOE, 2024)

Unit 6: Expressions and Equations

Unit Goal: In this unit, students learn to understand and use the terms “variable,” “coefficient,” “solution,” “equivalent expressions,” “exponent,” “independent variable,” and “dependent variable.” They begin to write coefficients next to variables without a multiplication symbol, e.g., $10x$ rather than $10 \cdot x$, and note that x is $x \cdot 1$. They learn other situations in which the multiplication symbol can be omitted, e.g., $6(3+2)$ can be written $6(3+2)$. They work with expressions that have positive whole-number exponents and whole-number, fraction, or variable bases, using properties of exponents strategically to evaluate these expressions, given a value for the variable. They find solutions for linear equations in one variable and simple equations that include exponents, e.g., $2^x=32$ and $100=x^2$. They use these terms and representations (including expressions with two variables) in reasoning about real-world and geometrical situations, understanding that some values of variables may not make sense in a given context. They represent collections of equivalent ratios as equations and use and make connections between tables, graphs, and linear equations that represent the same relationships.

Duration: 5 weeks

Essential Questions:

- How do variables, coefficients, and exponents contribute to understanding and solving mathematical expressions and equations?
- What are the roles of independent and dependent variables in mathematical models, and how do they relate to real-world scenarios?

- How can we manipulate expressions and equations using properties of exponents to simplify and solve problems?
- What strategies can we use to find solutions for linear equations in one variable and equations involving exponents?
- How do we represent and interpret real-world situations using mathematical expressions, equations, tables, and graphs?
- What connections exist between equivalent ratios, equations, tables, and graphs, and how can they be used to solve problems and make predictions?

Enduring Understandings:

- Variables, coefficients, and exponents are essential components of mathematical expressions and equations, allowing us to represent and analyze relationships between quantities.
- Understanding the roles of independent and dependent variables helps us create mathematical models that reflect real-world scenarios and make predictions based on given conditions.
- Manipulating expressions and equations using properties of exponents allows us to simplify complex expressions and solve equations efficiently.
- Solving linear equations in one variable and equations involving exponents requires applying appropriate strategies and understanding the relationship between variables and constants.
- Representing real-world situations using mathematical representations such as tables, graphs, and equations enables us to analyze data, make predictions, and solve problems in context.
- Connections between equivalent ratios, equations, tables, and graphs provide multiple perspectives for understanding relationships and solving problems across different mathematical domains.

Key NJSLS Strands:

- **6.EE.A** Apply and extend previous understandings of arithmetic to algebraic expressions
 - **6.EE.A.1** Write and evaluate numerical expressions involving whole-number exponents.
 - **6.EE.A.2** Write, read, and evaluate expressions in which letters stand for numbers.
 - **6.EE.A.2.a** Write expressions that record operations with numbers and with letters standing for numbers.
 - **6.EE.A.2.b** Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity.
 - **6.EE.2c** Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). For example, use the formulas and to find the volume and surface area of a cube with sides of length .

- **6.EE.A.3** Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression to produce the equivalent expression apply the distributive property to the expression to produce the equivalent expression ; apply properties of operations to to produce the equivalent expression .
- **6.EE.A.4** Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them).
- **6.EE.B** Reason about and solve one-variable equations and inequalities
 - **6.EE.B.5** Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
 - **6.EE.B.6** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
 - **6.EE.B.7** Solve real-world and mathematical problems by writing and solving equations of the form and for cases in which , and are all nonnegative rational numbers. *Climate Change Example: Students may solve real-world problems by writing and solving one-variable equations related to deforestation and/or increasing livestock farming as contributors to climate change.*
- **6.EE.C Represent and analyze quantitative relationships between dependent and independent variables**
 - **6.EE.C.9** Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *Climate Change Example: Students may analyze and use variables to represent the relationship between greenhouse emissions and livestock farming when representing relationships among contributors to climate change.*
- **6.RP.A** Understand ratio concepts and use ratio reasoning to solve problems
 - **6.RP.A.1** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”
 - **6.RP.A.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

- **6.RP.A.3a** Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
 - **6.RP.A.3.b** Solve unit rate problems including those involving unit pricing and constant speed.
 - **6.RP.A.3.c** Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means times the quantity); solve problems involving finding the whole, given a part and the percent.
- **6.NS.B** Compute fluently with multi-digit numbers & find common factors & multiple
 - **6.NS.B.2** With accuracy and efficiency, divide multi-digit numbers using the standard algorithm.
 - **6.NS.B.3** With accuracy and efficiency, add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Interdisciplinary Connections

- **NJSLSA.R7** Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words.
- **NJSLSA.SL1** Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively
- **NJSLSA.SL4** Present information, findings, and supporting evidence such that listeners can follow the line of reasoning and the organization, development, and style are appropriate to task, purpose, and audience.
- **MS-ETS1-1** Students estimate sums and differences while defining the criteria and constraints of a design problem with sufficient precision to ensure a successful solution, taking into account relevant scientific principles.

21st-Century Skills

- **9.4.8.CT.2** Develop multiple solutions to a problem and evaluate short- and long-term effects to determine the most plausible option).

Career Ready Practices

- **CRP2** Apply appropriate academic and technical skills.

Technology Integration
● 9.4.5.CT.3 Describe how digital tools and technology may be used to solve problems.

Skills	Instructional Topics	Assessment	Resources	Vocabulary
Write a numerical expression using whole-number exponents. Evaluate numerical expressions involving whole number exponents. Determine if a given number from a specified set is a solution to an equation or an inequality using substitution. Write and solve equations of the form $x + p = q$ and $px = q$, where p, q, and x are all nonnegative rational numbers, for real-world and mathematical problems.	Equations in One Variable Tape Diagrams and Equations Truth and Equations Staying in Balance Practice Solving Equations and Representing Situations with Equations A New Way to Interpret a over b Equal and Equivalent Write Expressions Where Letters Stand for Numbers Revisit Percentages Equal and Equivalent The Distributive	Formative Assessments: Diagnostic Assessment Mid-topic Checkpoint Lesson Quizzes Lesson Self-assessment Teacher Observations Class Discussions Classwork/ Homework Exit Slips/Google Form Summative Assessments: Topic/Unit Assessments Constructed Response Questions Projects	Materials: 1/2-inch cubes 1/2-inch graph paper base-ten blocks beakers bingo chips blank paper colored pencils cuisenaire rods decks of playing cards demonstration nets with and without flaps dot stickers empty containers food coloring four-function calculators	value (of a variable) operation variable Coefficient solution solution to an equation true equation / false equation each side balanced hanger solve (an equation) equation true equation / false equation

Variables are used to represent unknown numbers, including any number in a specified set. Explain that two quantities which change in relationship to one another are expressed as independent and dependent variables. Write an equation using two quantities, an independent and a dependent variable, to represent a real-world problem. Analyze the relationship between the dependent and independent variables using graphs and tables and relate them to the equation Write coefficients next to variables without a multiplication symbol, e.g., $10x$ rather than $10 \cdot x$, and note that x is	Property Meaning of Exponents Expressions with Exponents Evaluating Expressions with Exponents Equivalent Exponential Expressions Relationships Between Quantities Two Related Quantities More Relationships Tables, Equations, and Graphs	Cumulative Tests Performance Assessments Alternate and Benchmark Assessments: Progress Monitoring Assessments Cumulative/Benchmark Assessments	gallon-sized jug tracing paper graph paper colored pencils scissors index cards ruler protractor glue or glue sticks graduated cylinders hand-held graphing calculator graph paper grocery store circulars household items inch cubes index cards internet-enabled device liter-sized bottle	equivalent expressions term distributive property area as a product area as a sum to the power base (of an exponent) exponent solution to an equation independent variable dependent variable variable relationship coordinate plane coordinates horizontal axis vertical axis
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<i>I.x.</i> Learn other situations in which the multiplication symbol can be omitted, e.g., can be written Work with expressions that have positive whole-number exponents and whole-number, fraction, or variable bases, using properties of exponents strategically to evaluate these expressions, given a value for the variable. Find solutions for linear equations in one variable and simple equations that include exponents, e.g., Use these terms and representations (including expressions with two variables) in reasoning about real-world and geometrical situations, understanding that some values of variables may not make sense in a			markers masking tape measuring tapes metal paper fasteners (brass brads) meter sticks nets of polyhedra origami paper paper clips paper cups pattern blocks pre-assembled or commercially produced polyhedra pre-assembled or commercially produced tangrams quart-sized bottle rulers rulers marked with centimeters	plot
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given context. Represent collections of equivalent ratios as equations and use and make connections between tables, graphs, and linear equations that represent the same relationships. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient). View one or more parts of an expression as a single entity. Perform arithmetic operations, utilizing the Order of Operations, that include whole number exponents and no parentheses.			salt scale a digital scale that can output in grams, kilograms, ounces, or pounds scissors snap cubes sticky notes stopwatches straightedges swtring tape teaspoon tools for creating a visual display tracing paper tray water yardsticks Activities:	
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Identify when two expressions are equivalent. Generate equivalent expressions using the properties of operations Explain that two expressions are equivalent when they name the same number regardless of which value is substituted into them. Write an algebraic expression from a verbal description that includes operations, numbers, and variables. Write expressions using variables to represent real-world or mathematical situations. Evaluate expressions, including formulas, for			Variable Voyages Diagram Detective Equation Balance Challenge Balancing Act Adventures Equation Expedition Fraction Frontier Equation Equality Quest Expression Exploration Percentage Puzzle Party Distributive Delight Exponential Explorers Exponent Expedition Exponent Evaluation Excursion Exponential Equivalence Expedition Quantity Quest Twin Quantity Trek	
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specific values of the variables.			Quantity Connection Challenge Data Discovery Dash Mentor Texts: <i>The Curious Incident of the Dog in the Night-Time</i> by Mark Haddon (While not explicitly about mathematics, this novel features a protagonist who uses logical reasoning and problem-solving skills to navigate the world, demonstrating the importance of critical thinking in solving complex problems.)	
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Modifications and Accommodations	
Tier II and III Strategies	Small group instruction (Tier II) ● Explicit instruction ● Schema-based instruction ● Use visuals ● Use manipulatives ● Use a multi-sensory approach ● Guided notes and/or scaffold outline for written assignments ● Introduce key vocabulary before lesson

	• Lesson taught again using a differentiated approach Small group or Individual instruction (Tier III) • Intensive instruction • Practice fact retrieval • Lesson taught again using a differentiated approach • Allow answers to be given orally or dictated • Use visuals / Anchor Charts
Students with IEPs and 504 Plans	<u>Universal Design for Learning and Access for Students with Disabilities</u> <u>Movematics (dyscalculia.org)</u> Follow all the IEP/504 plan modifications Allow extra time to complete assignments or tests Guided notes and/or scaffold outline for written assignments Work in a small group Solidify and refine concepts through repetition Allow answers to be given orally or dictated Use multi-sensory teaching approaches Utilize assistive technology and materials Use large print books, Braille, or books on CD (digital text) Text to speech/audio recorded selections Amplification system as needed Leveled texts according to ability

	Fine motor skill stations embedded in rotation as needed Modified or constrained spelling word lists Provide anchor charts with high frequency words and phonemic patterns Students might need guidance as to what actions happen during partner work time. Invite students to discuss what it looks like when they are working with their partner while exploring the connecting cubes. Generate a list of shared expectations for partner work. Invite students to plan a strategy before they begin working. If time allows, invite students to share their plan with a partner. Some students may benefit from feedback that emphasizes effort and time on task. For example, moving counters in the squares of the 5-frame or filling the 5-frame with one counter in each square.
Multi-language Learners	<u>Mathematical Language Development and Access for English Learners</u> Work toward longer passages as skills in English increase Use visuals Introduce/highlight key vocabulary before lesson Teacher models reading aloud daily Provide peer tutoring Utilize translation supports including bilingual dictionary, translation device, assistive technology, and peers Guided notes and/or scaffold outline for written assignments Provide students with English Learner leveled readers

	Support recommended by the grade specific “can do” list outlined by WIDA, including Key Uses Edition and Descriptor Name Charts Create a visual display of what students want to do or make with the connecting cubes. As students share ideas, include a drawing or annotation to illustrate connections. Invite students to use gestures as they count.
Accelerating Students	<u>Accelerating Students Guide</u> Create an enhanced set of introductory activities (e.g. advance organizers, concept maps, concept puzzles) Provide options, alternatives and choices to differentiate and broaden the curriculum Organize and offer flexible small group learning activities Provide whole group enrichment explorations Teach cognitive and methodological skills Use center, stations, or contracts Organize integrated problem-solving simulations Propose interest-based extension activities Expose students to beyond level texts.

(DOE, 2024)

Unit 7: Rational Numbers

Unit Goal: In this unit, students interpret signed numbers in contexts (e.g., temperature above or below zero, elevation above or

below sea level). They understand and use the terms “positive number,” “negative number,” “rational number,” “opposite,” “sign,” “absolute value,” “a solution to an inequality,” “less than,” “greater than,” and the corresponding symbols. They plot points with signed rational number coordinates on the number line, and recognize and use the connection between relative position of two points on the number line and inequalities involving the coordinates of the points. (These are limited to strict inequalities rather than inequalities such as $\leq$ which occur in grade 7.) They understand and use absolute value notation, understanding that the absolute value of a number is its distance from zero on the number line. Students graph inequalities in one variable on number line diagrams, using a circle or disk to indicate when a given point is, respectively, excluded or included. They solve simple inequalities, understanding that there may be infinitely many solutions, and show solutions symbolically and on the number line. They interpret solutions of inequalities in contexts, understanding that some solutions do not make sense in some contexts. Students plot pairs of signed number coordinates in the plane, understanding the relationship between the signs of a pair of coordinates and the quadrant of the corresponding point, and use coordinates to calculate horizontal and vertical distances between two points. Students understand and use the terms “common factor,” “greatest common factor,” “common multiple,” and “least common multiple,” and solve problems set in real-world contexts in which common factors or multiples occur.

Duration: 4 weeks

Essential Questions:

- What are integers?
- How can you plot rational numbers on a number line?
- How can you compare and order rational numbers?
- How can you represent and interpret absolute values?

Enduring Understandings:

- Integers are the counting numbers, their opposites, and zero.
- Each rational number can be associated with a unique point on the number line.
- Compare and order rational numbers
- Interpret absolute value in mathematics and real-world situations.

Key NJSLS Strands:

- **6.NS.A** Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
 - **6.NS.A.1** Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.
- **6.NS.B** Compute fluently with multi-digit numbers & find common factors & multiples

- **6.NS.B.4** Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor.
- **6.NS.C** Apply and extend previous understandings of numbers to the system of rational numbers
 - **6.NS.C.5** Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
 - **6.NS.C.6** Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.
 - **6.NS.C.6.a** Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., , and that 0 is its own opposite.
 - **6.NS.C.6.b** Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
 - **6.NS.C.6.c** Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
 - **6.NS.C.7** Understand ordering and absolute value of rational numbers.
 - **6.NS.C.7a** Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.
 - **6.NS.C.7.b** Write, interpret, and explain statements of order for rational numbers in real-world contexts.
 - **6.NS.C.7c** Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. For example, for an account balance of dollars, write to describe the size of the debt in dollars.
 - **6.NS.C.7.d** Distinguish comparisons of absolute value from statements about order. For example, recognize that an account balance less than dollars represents a debt greater than 30 dollars.
- **6.NS.C.8** Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.
- **6.EE.B** Reason about and solve one-variable equations and inequalities

- **6.EE.B.5** Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
- **6.EE.B.6** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
- **6.EE.B.7** Solve real-world and mathematical problems by writing and solving equations of the form $ax + b = c$ and for cases in which a , b , and c are all nonnegative rational numbers. **Climate Change Example:** Students may solve real-world problems by writing and solving one-variable equations related to deforestation and/or increasing livestock farming as contributors to climate change.
- **6.EE.B.8** Write an inequality of the form $ax + b < c$ or $ax + b > c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $ax + b < c$ or $ax + b > c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.
- **6.EE.C Represent and analyze quantitative relationships between dependent and independent variables**
 - **6.EE.C.9** Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. **Climate Change Example:** Students may analyze and use variables to represent the relationship between greenhouse emissions and livestock farming when representing relationships among contributors to climate change.
 - **6.G.A** Solve real-world and mathematical problems involving area, surface area, and volume.
- **6.G.A.3** Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

Interdisciplinary Connections

- **ELA: W.6.1** When students are justifying their reasoning on short answer and extended constructed response questions they write arguments to support claims with clear reasons and relevant evidence.
- **MS-ESS1-3** When students are creating and comparing ratios they analyze and interpret data to determine scale properties of objects in the solar system

21st-Century Skills
● 9.2.8.CAP.12 Assess personal strengths, talents, values, and interests to appropriate jobs and careers to maximize career potential.
Career Ready Practices
● CRP2 Apply appropriate academic and technical skills. ● CRP8 Utilize critical thinking to make sense of problems and persevere in solving them. ● CRP11 Use technology to enhance productivity.
Technology Integration
● 9.4.8.TL.3 Select appropriate tools to organize and present information digitally. ● 9.4.5.CT.3 Describe how digital tools and technology may be used to solve problems.

Skills	Instructional Topics	Assessment	Resources	Vocabulary
Explain the signs of an ordered pair indicate its quadrant location in the coordinate plane Understand that ordered pairs that differ only by signs are reflections across one or both axes Locate numbers with opposite signs as points on opposite sides of zero on the number line	Negative Numbers and Absolute Value Positive and Negative Numbers Points on the Number Line Comparing Positive and Negative Numbers Ordering Rational Numbers	Formative Assessments: Diagnostic Assessment Mid-topic Checkpoint Lesson Quizzes Lesson Self-assessment Teacher Observations Class Discussions	Materials: 1/2-inch cubes 1/2-inch graph paper base-ten blocks beakers bingo chips blank paper colored pencils	positive number negative number distance (away) from zero closer to 0 farther from 0 maximum minimum

Explain that the opposite of an opposite of a number is the number itself and that zero is its own opposite Represent the relative position of two numbers on a number line diagram using inequality statements Write and interpret statements of order using rational numbers to explain real-world problems Demonstrate knowledge that the absolute value of a rational number is its distance from zero on the number line Express the magnitude of a positive or negative quantity in a real-world situation using absolute value Know that statements about order are used to distinguish comparisons	Using Negative Numbers to Make Sense of Contexts Absolute Value of Numbers Comparing Numbers and Distance from Zero Inequalities Writing and Graphing Inequalities Solutions of Inequalities Interpreting Inequalities The Coordinate Plane Points on the Coordinate Plane Constructing the Coordinate Plane Interpreting Points on a Coordinate Plane Distances on a Coordinate Plane Shapes on the Coordinate Plane	Classwork/ Homework Exit Slips/Google Form Summative Assessments: Topic/Unit Assessments Constructed Response Questions Projects Cumulative Tests Performance Assessments Alternate and Benchmark Assessments: Progress Monitoring Assessments Cumulative/Benchmark Assessments	cuisenaire rods decks of playing cards demonstration nets with and without flaps dot stickers empty containers food coloring four-function calculators gallon-sized jug tracing paper graph paper colored pencils scissors index cards ruler protractor glue or glue sticks graduated cylinders hand-held graphing calculator	requirement solution to an inequality unbalanced hanger inequality quadrant (line) segment axis degrees Fahrenheit degrees Celsius absolute value x-coordinate y-coordinate factor common factor greatest common factor (GCF) multiple common multiple least common multiple (LCM)
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of absolute value Represent a constraint or condition in a real-world or mathematical problem by writing an inequality in the form $x > c$ or $x < c$ Explain inequalities of the form $x > c$ or $x < c$ have infinitely many solutions Represent the infinitely many solutions to the inequalities $x > c$ or $x < c$ on a number line diagram	Common Factors Using Common Multiples and Common Factors Drawing on the Coordinate Plane		graph paper grocery store circulars household items inch cubes index cards internet-enabled device liter-sized bottle markers masking tape measuring tapes metal paper fasteners (brass brads) meter sticks nets of polyhedra origami paper paper clips paper cups pattern blocks	
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			pre-assembled or commercially produced polyhedra pre-assembled or commercially produced tangrams quart-sized bottle rulers rulers marked with centimeters salt scale a digital scale that can output in grams, kilograms, ounces, or pounds scissors snap cubes sticky notes stopwatches straightedges swtring	
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			tape teaspoon tools for creating a visual display tracing paper tray water yardsticks Activities: Number Line Walk Positive/Negative Number Sort Human Number Line Number Line Relay Interactive Number Line Exploration Number Line Battleship Number War Rational Number Sort	
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			Temperature Scenarios Temperature Tracker Rational Number Line Up Rational Number Puzzle Budgeting Challenge Elevation Exploration Mentor Texts: <i>The Phantom Tollbooth</i> by Norton Juster	
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Modifications and Accommodations	
Tier II and III Strategies	Small group instruction (Tier II) ● Explicit instruction ● Schema-based instruction ● Use visuals ● Use manipulatives ● Use a multi-sensory approach ● Guided notes and/or scaffold outline for written assignments ● Introduce key vocabulary before lesson ● Lesson taught again using a differentiated approach Small group or Individual instruction (Tier III) ● Intensive instruction ● Practice fact retrieval

	● Lesson taught again using a differentiated approach ● Allow answers to be given orally or dictated ● Use visuals / Anchor Charts
Students with IEPs and 504 Plans	<u>Universal Design for Learning and Access for Students with Disabilities</u> <u>Movematics (dyscalculia.org)</u> Follow all the IEP/504 plan modifications Allow extra time to complete assignments or tests Guided notes and/or scaffold outline for written assignments Work in a small group Solidify and refine concepts through repetition Allow answers to be given orally or dictated Use multi-sensory teaching approaches Utilize assistive technology and materials Use large print books, Braille, or books on CD (digital text) Text to speech/audio recorded selections Amplification system as needed Leveled texts according to ability Fine motor skill stations embedded in rotation as needed Modified or constrained spelling word lists

	Provide anchor charts with high frequency words and phonemic patterns Students might need guidance as to what actions happen during partner work time. Invite students to discuss what it looks like when they are working with their partner while exploring the connecting cubes. Generate a list of shared expectations for partner work. Invite students to plan a strategy before they begin working. If time allows, invite students to share their plan with a partner. Some students may benefit from feedback that emphasizes effort and time on task. For example, moving counters in the squares of the 5-frame or filling the 5-frame with one counter in each square.
Multi-language Learners	<u>Mathematical Language Development and Access for English Learners</u> Work toward longer passages as skills in English increase Use visuals Introduce/highlight key vocabulary before lesson Teacher models reading aloud daily Provide peer tutoring Utilize translation supports including bilingual dictionary, translation device, assistive technology, and peers Guided notes and/or scaffold outline for written assignments Provide students with English Learner leveled readers Support recommended by the grade specific “can do” list outlined by WIDA, including Key Uses Edition and Descriptor Name Charts

	Create a visual display of what students want to do or make with the connecting cubes. As students share ideas, include a drawing or annotation to illustrate connections. Invite students to use gestures as they count.
Accelerating Students	<u>Accelerating Students Guide</u> Create an enhanced set of introductory activities (e.g. advance organizers, concept maps, concept puzzles) Provide options, alternatives and choices to differentiate and broaden the curriculum Organize and offer flexible small group learning activities Provide whole group enrichment explorations Teach cognitive and methodological skills Use center, stations, or contracts Organize integrated problem-solving simulations Propose interest-based extension activities Expose students to beyond level texts.

(DOE, 2024)

Unit 8: Data Sets and Distributions

Unit Goal: In this unit, students learn about populations and study variables associated with a population. They understand and use the terms “numerical data,” “categorical data,” “survey” (as noun and verb), “statistical question,” “variability,” “distribution,” and “frequency.” They make and interpret histograms, bar graphs, tables of frequencies, and box plots. They describe distributions

(shown on graphical displays) using terms such as “symmetrical,” “peaks,” “gaps,” and “clusters.” They work with measures of center—understanding and using the terms “mean,” “average,” and “median.” They work with measures of variability—understanding and using the terms “range,” “mean absolute deviation” or MAD, “quartile,” and “interquartile range” or IQR. They interpret measurements of center and variability in contexts.

Duration: 4 weeks

Essential Questions:

- How can you identify and write statistical questions?
- How can you use a single number to describe a data set?
- How can the mean, median and mode be used to summarize data?
- How can you make and use a frequency table and a histogram?
- How can variability of data be described with one number?
- Which statistical measure is most useful to describe a given situation?
- How can a data distribution be summarized?

Enduring Understandings:

- Statistical questions anticipate variability in responses and can be answered by collecting and analyzing data.
- The mean is one measure of center used to describe a center of data.
- Different measures can be used to describe a set of data.
- Data can be organized into equal ranges using a frequency table or a histogram.
- Measures of Variability describe the spread and clustering of data in a set.
- Measures of center can be used to describe and summarize a data set.
- A set of data collected to answer a statistical question has a distribution that can be described by its center, spread and overall shape.

Key NJSL Standards:

- **6.SP.A** Develop understanding of statistical variability
 - **6.SP.A.1** Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. For example, “How old am I?” is not a statistical question, but “How old are the students in my school?” is a statistical question because one anticipates variability in students’ ages.

- **6.SP.A.2** Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.
- **6.SP.A.3** Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.
- **6.SP.B** Summarize and describe distributions
 - **6.SP.B.4** Display numerical data in plots on a number line, including dot plots, histograms, and box plots. *Climate Change Example: Students may display numerical data related to deforestation and increasing livestock farming as contributors to climate change in plots on a number line, including dot plots, histograms, and box plots.*
 - **6.SP.B.5** Summarize numerical data sets in relation to their context, such as by:
 - **6.SP.B.5a** Reporting the number of observations.
 - **6.SP.B.5.b** Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
 - **6.SP.B.5.c** Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
 - **6.SP.B.5.d** Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.
- **6.NS.B** Compute fluently with multi-digit numbers & find common factors & multiples
 - **6.NS.B.2** With accuracy and efficiency, divide multi-digit numbers using the standard algorithm.
 - **6.NS.B.3** With accuracy and efficiency, add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.
 - **6.NS.B.4** Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor.

Interdisciplinary Connections

- **NJSLSA.R7** Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words.

● NJSLSA.SL1 Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively. ● NJSLSA.SL4 Present information, findings, and supporting evidence such that listeners can follow the line of reasoning and the organization, development, and style are appropriate to task, purpose, and audience. ● MS-LS2-2 In this statistics unit students construct an explanation that predicts patterns of interactions among organisms.
21st-Century Skills
● 9.1.8.FP.2 Evaluate the role of emotions, attitudes, and behavior (rational and irrational) in making financial decisions.
Career Ready Practices
● CRP2 Apply appropriate academic and technical skills. ● CRP8 Utilize critical thinking to make sense of problems and persevere in solving them. ● CRP11 Use technology to enhance productivity.
Technology Integration
● 9.4.8.TL.6 Collaborate to develop and publish work that provides perspectives on a real-world problem.

Skills	Instructional Topics	Assessment	Resources	Vocabulary
Define a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. Recognize statistical questions.	Data, Variability, and Statistical Questions Data Statistical Questions, Dot Plots, and Histograms Representing Data	Formative Assessments: Diagnostic Assessment Mid-topic Checkpoint Lesson Quizzes Lesson Self-assessment	Materials: 1/2-inch cubes 1/2-inch graph paper base-ten blocks beakers bingo chips	numerical data categorical data dot plot statistical question variability distribution

Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. Determine that a measure of center (mean and median) for a numerical data set summarizes all of its values with a single number. Recognize that a measure of variation (interquartile range and mean absolute deviation) describes how its values vary with a single number. Display numerical data in plots on a number line, including dot plots, histograms, and box plots Summarize numerical data sets in relation to their context, such as by reporting the number of	Graphically Dot Plots Using Dot Plots to Answer Statistical Questions Interpreting Histograms Using Histograms to Answer Statistical Questions Describing Distributions on Histograms Measures of Center and Variability Mean Finding and Interpreting the Mean as the Balance Point Variability and MAD Using Mean and MAD to Make Comparisons Median and IQR Median Comparing Mean and	Teacher Observations Class Discussions Classwork/ Homework Exit Slips/Google Form Summative Assessments: Topic/Unit Assessments Constructed Response Questions Projects Cumulative Tests Performance Assessments Alternate and Benchmark Assessments: Progress Monitoring Assessments Cumulative/Benchmark Assessments	blank paper colored pencils cuisenaire rods decks of playing cards demonstration nets with and without flaps dot stickers empty containers food coloring four-function calculators gallon-sized jug tracing paper graph paper colored pencils scissors index cards ruler protractor glue or glue sticks	frequency bar graph typical center spread variability histogram bins distribution center statistical question spread symmetrical peak cluster unusual value gap average mean
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observations and describing how it was measured and the units for the measurement. Describe overall patterns and any striking deviations from a data set by giving the measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation) with reference to the context with which the data was collected. Understand that the shape of the data distribution and the context in which the data were gathered can be related to the choice of measures of center and variability. Divide multi-digit numbers using the standard algorithm working towards accuracy and efficiency.	Median Quartiles and Interquartile Range Box Plots Using Box Plots Using Data to Solve Problems		graduated cylinders hand-held graphing calculator graph paper grocery store circulars household items inch cubes index cards internet-enabled device liter-sized bottle markers masking tape measuring tapes metal paper fasteners (brass brads) meter sticks nets of polyhedra origami paper paper clips paper cups	fair share measure of center balance point mean absolute deviation (MAD) measure of spread symmetrical typical median peak cluster range quartile interquartile range (IQR) five-number summary measure of spread minimum maximum box plot
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Add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation, working towards accuracy and efficiency. Find the greatest common factor of two whole numbers that are less than or equal to 100. Find the least common multiple of two whole numbers that are less than or equal to 12. Use the distributive property to factor the greatest common factor from a sum of two whole numbers in the range 1 to 100.			pattern blocks pre-assembled or commercially produced polyhedra pre-assembled or commercially produced tangrams quart-sized bottle rulers rulers marked with centimeters salt scale a digital scale that can output in grams, kilograms, ounces, or pounds scissors snap cubes sticky notes stopwatches straightedges	whisker median
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			swtring tape teaspoon tools for creating a visual display tracing paper tray water yardsticks Activities: GCF and LCM Exploration Distributive Property Activity Real-Life Scenario Data Analysis Activity Data Representation Gallery Walk Problem-Solving Task Cards Error Analysis Factors and Multiples	
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			Factor Tree Exploration LCM Challenge Mentor Texts: <i>The Boy Who Loved Math: The Improbable Life of Paul Erdős</i> by Deborah Heiligman	
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Modifications and Accommodations	
Tier II and III Strategies	Small group instruction (Tier II) • Explicit instruction • Schema-based instruction • Use visuals • Use manipulatives • Use a multi-sensory approach • Guided notes and/or scaffold outline for written assignments • Introduce key vocabulary before lesson • Lesson taught again using a differentiated approach Small group or Individual instruction (Tier III) • Intensive instruction • Practice fact retrieval • Lesson taught again using a differentiated approach • Allow answers to be given orally or dictated • Use visuals / Anchor Charts
Students with IEPS and 504 Plans	<u>Universal Design for Learning and Access for Students with Disabilities</u>

[Movematics \(dyscalculia.org\)](http://dyscalculia.org)

Follow all the IEP/504 plan modifications

Allow extra time to complete assignments or tests

Guided notes and/or scaffold outline for written assignments

Work in a small group

Solidify and refine concepts through repetition

Allow answers to be given orally or dictated

Use multi-sensory teaching approaches

Utilize assistive technology and materials

Use large print books, Braille, or books on CD (digital text)

Text to speech/audio recorded selections

Amplification system as needed

Leveled texts according to ability

Fine motor skill stations embedded in rotation as needed

Modified or constrained spelling word lists

Provide anchor charts with high frequency words and phonemic patterns

Students might need guidance as to what actions happen during partner work time. Invite students to discuss what it looks like when they are working with their partner while exploring the connecting cubes. Generate a list of shared expectations for partner work.

Invite students to plan a strategy before they begin working. If time allows, invite

	students to share their plan with a partner. Some students may benefit from feedback that emphasizes effort and time on task. For example, moving counters in the squares of the 5-frame or filling the 5-frame with one counter in each square.
Multi-language Learners	<u>Mathematical Language Development and Access for English Learners</u> Work toward longer passages as skills in English increase Use visuals Introduce/highlight key vocabulary before lesson Teacher models reading aloud daily Provide peer tutoring Utilize translation supports including bilingual dictionary, translation device, assistive technology, and peers Guided notes and/or scaffold outline for written assignments Provide students with English Learner leveled readers Support recommended by the grade specific “can do” list outlined by WIDA, including Key Uses Edition and Descriptor Name Charts Create a visual display of what students want to do or make with the connecting cubes. As students share ideas, include a drawing or annotation to illustrate connections. Invite students to use gestures as they count.
Accelerating Students	<u>Accelerating Students Guide</u>

	Create an enhanced set of introductory activities (e.g. advance organizers, concept maps, concept puzzles) Provide options, alternatives and choices to differentiate and broaden the curriculum Organize and offer flexible small group learning activities Provide whole group enrichment explorations Teach cognitive and methodological skills Use center, stations, or contracts Organize integrated problem-solving simulations Propose interest-based extension activities Expose students to beyond level texts.
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(DOE, 2024)

Unit 9: Integrating Math

Unit Goal: In this unit, students use concepts and skills from previous units. In solving Fermi problems, they use measurement conversions together with their knowledge of volumes or surface areas of right rectangular prisms or the relationship of distance, rate, and time. In answering questions about ratios of two populations, they work with percentages that include numbers expressed in the form $\frac{a}{b}$ or as decimals. In answering questions about diagrams of rectangles with whole-number dimensions, they connect arithmetic features of the dimensions such as remainder or greatest common factor with geometric features of the diagrams. In answering questions about votes, voting methods, and equitable distribution, they use their knowledge of equivalent ratios, part-part ratios, percentages, and unit rates.

Duration: 2 weeks

Essential Questions:

- How can measurement conversions and geometric concepts be applied to solve real-world Fermi problems effectively?
- What role do ratios and percentages play in understanding and analyzing populations and voting data?
- How do arithmetic and geometric features interact in diagrams of rectangles, and how does this understanding contribute to problem-solving?
- How can knowledge of equivalent ratios, part-part ratios, percentages, and unit rates inform equitable distribution in voting methods and other scenarios?

Enduring Understandings:

- Measurement conversions and geometric concepts are powerful tools for solving complex problems, such as Fermi problems, by providing a framework for organizing and manipulating data effectively.
- Ratios and percentages are fundamental mathematical concepts that allow us to analyze and compare populations, voting data, and other quantities, providing insights into relationships and trends.
- The relationship between arithmetic features (e.g., remainders, greatest common factors) and geometric features (e.g., dimensions, area) of diagrams of rectangles illustrates the interconnectedness of mathematical concepts and their application to problem-solving.
- Knowledge of equivalent ratios, part-part ratios, percentages, and unit rates enables informed decision-making and equitable distribution in various contexts, including voting methods and resource allocation.

Key NJSLS Strands:

- **6.G.A** Solve real-world and mathematical problems involving area, surface area, and volume.
- **6.NS.A** Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
- **6.NS.B** Compute fluently with multi-digit numbers & find common factors & multiples
 - **6.NS.B.3** With accuracy and efficiency, add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.
- **6.RP.A** Understand ratio concepts and use ratio reasoning to solve problems
 - **6.RP.A.1** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”
 - **6.RP.A.2** Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. For example, “This recipe has a

ratio of 3 cups of flour to 4 cups of sugar, so there is $\frac{3}{4}$ -cup of flour for each cup of sugar.” “We paid \$75 for 15 hamburgers, which is a rate of \$5 per hamburger.”
(Clarification: Expectations for unit rates in this grade are limited to non-complex fractions.)

- **6.RP.A.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
 - **6.RP.A.3.c** Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means times the quantity); solve problems involving finding the whole, given a part and the percent.

Interdisciplinary Connections

- **NJSLSA.R7** Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words.
- **NJSLSA.SL1** Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others’ ideas and expressing their own clearly and persuasively
- **NJSLSA.SL4** Present information, findings, and supporting evidence such that listeners can follow the line of reasoning and the organization, development, and style are appropriate to task, purpose, and audience.
- **MS-ETS1-1:** Students estimate sums and differences while defining the
- criteria and constraints of a design problem with sufficient precision to ensure a
- successful solution, taking into account relevant scientific principles.

21st-Century Skills

- **9.1.8.CR.2** Compare various ways to give back through strengths, passions, goals, and other personal factors.

Career Ready Practices

- **CRP2** Apply appropriate academic and technical skills. CRP4: Communicate clearly and effectively with reason. CRP6: Demonstrate creativity and innovation.

Technology Integration

- **9.4.5.CT.3** Describe how digital tools and technology may be used to solve problems.

Skills	Instructional Topics	Assessment	Resources	Vocabulary
Apply the techniques of finding area, surface area, and volumes of right rectangular prisms with fractional edge lengths to solve real-world and mathematical problems. Compute quotients of fractions. Interpret quotients of fractions. Solve word problems involving. Add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation, working towards accuracy and	Making Connections Fermi Problems If Our Class Were the World Rectangle Madness Voting How Do We Choose? More than Two Choices Picking Representatives	Formative Assessments: Diagnostic Assessment Mid-topic Checkpoint Lesson Quizzes Lesson Self-assessment Teacher Observations Class Discussions Classwork/ Homework Exit Slips/Google Form Summative Assessments: Topic/Unit Assessments	Materials: 1/2-inch cubes 1/2-inch graph paper base-ten blocks beakers bingo chips blank paper colored pencils cuisenaire rods decks of playing cards demonstration nets with and without flaps dot stickers empty containers	mixed number in favor majority plurality runoff majority in all fair

efficiency. Explain the concept of a ratio through definition. Use ratio language to describe a relationship between two quantities. Construct a unit rate (a/b) from a given ratio (a:b). Explain a unit rate (a/b) associated with a ratio (a:b). Express a ratio relationship using rate language. Represent and solve rate and ratio real-world and mathematical problems by using tables, tape diagrams, double number line diagrams, and equations. Create tables of equivalent ratios and find missing values with whole number measurements.		Constructed Response Questions Projects Cumulative Tests Performance Assessments Alternate and Benchmark Assessments: Progress Monitoring Assessments Cumulative/Benchmark Assessments	food coloring four-function calculators gallon-sized jug tracing paper graph paper colored pencils scissors index cards ruler protractor glue or glue sticks graduated cylinders hand-held graphing calculator graph paper grocery store circulars household items inch cubes index cards	
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Plot pairs of values, in the coordinate plane, from a ratio table to compare ratios. Solve unit rate problems, including unit pricing and constant speed. Find the part, whole, and percent of a quantity in real-world problems. Unit ratios can be used to manipulate and transform units accurately. Convert measurement units utilizing ratio reasoning.			internet-enabled device liter-sized bottle markers masking tape measuring tapes metal paper fasteners (brass brads) meter sticks nets of polyhedra origami paper paper clips paper cups pattern blocks pre-assembled or commercially produced polyhedra pre-assembled or commercially produced tangrams quart-sized bottle rulers	
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			rulers marked with centimeters salt scale a digital scale that can output in grams, kilograms, ounces, or pounds scissors snap cubes sticky notes stopwatches straightedges swtring tape teaspoon tools for creating a visual display tracing paper tray water	
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			yardsticks Activities: Real-world Prism Problems Modeling with Fractional Prism Blocks Fraction Quotient Practice Word Problems with Fraction Quotients Decimal Algorithm Practice Real-world Decimal Word Problems Exploring Ratios with Manipulatives Ratio Problem-solving Stations Unit Rate Scavenger Hunt Percent of a Quantity Problems Mentor Texts:	
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			<i>Al Capone Does My Shirts</i> by Gennifer Choldenko (for unit rates and proportional reasoning)	
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(DOE, 2024)

Prepared by Julie Lang, MA

References

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NJ DOE. (2024). *New Jersey Mathematics 2023 Student Learning Standards*. Office of Standards, Division of Teaching and Learning Services, New Jersey Department of Education. <https://www.nj.gov/education/standards/math/Index.shtml>