

Englewood on the Palisades Charter School

Mathematics Curriculum

Grade 8

Aligned With the 2023 New Jersey Student Learning Standards

The Vision for Mathematics Education in New Jersey

A New Jersey education in Mathematics builds quantitatively and analytically literate citizens prepared to meet the demands of college and career, and to engage productively in an information-driven society. All students will have access to a high-quality mathematics education that fosters a population that:

- leverages data in decision-making and as a lens for discussing, analyzing, and responding to practical questions.
- persists to make sense of and model problems arising in everyday life, society, and the workplace.
- thinks critically and strategically to assess quantitative relationships and to solutions to complex problems.
- employs precise reasoning and constructs viable arguments to deduce conclusions, recognize false statements and assess peers' reasoning.
- interprets, evaluates and critiques the mathematics embedded in social, scientific and commercial systems, as well as the claims made in the private and public sectors.
- communicates precisely when conveying, representing, and justifying both qualitative and quantitative perspectives.

(NJ DOE, 2024)

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

Standard	Mathematically Proficient Students ...
1. Make sense of problems and persevere in solving them	start by explaining to themselves the meaning of a problem and looking for entry points to its solution. check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?”
2. Reason abstractly and quantitatively	make sense of quantities and their relationships in problem situations.
3. Construct viable arguments and critique the reasoning of others	understand and use stated assumptions, definitions, and previously established results in constructing arguments.
4. Model with mathematics	apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later.
5. Use appropriate tools strategically	are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations.
6. Attend to precision	try to communicate precisely to others.
7. Look for and make use of structure	look closely to discern a pattern or structure

8. Look for and express regularity in repeated reasoning	notice if calculations are repeated, and look both for general methods and for shortcuts.
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(NJ DOE, 2024)

New Jersey Legislative Statutes and Administrative Code Mathematics

Amistad Law: *N.J.S.A. 18A 52:16A-88*

[Radical Math](#): includes lessons on topics for social justice in mathematics
[African American Mathematician Bios](#)

Holocaust Law: *N.J.S.A. 18A:35-28*

<https://www.bbc.com/future/article/20191031-hilda-geiringer-mathematician-who-fled-the-nazis>

LGBT and Disabilities Law: *N.J.S.A. 18A:35-4.35*

<https://prideinstem.org/>
[LGBTQA+ Mathematician Bios](#)
<https://www.ngpf.org/blog/math/math-monday-celebrating-disabled-mathematicians/>
[Mathematicians with Disabilities](#)

Diversity & Inclusion: *N.J.S.A. 18A:35-4.36a*

<http://www.ams.org/about-us/diversity>
<https://mathematicallygiftedandblack.com/>

Standards in Action: <i>Climate Change</i>
New Jersey became the first state in the nation to include climate change across content areas with the adoption of the 2020 New Jersey Student Learning Standards (NJSLS). The goal of inclusion of climate change education implementation is to foster generations of New Jersey students that can analyze, question, interpret, to think independently, and bring critical deduction to fulfill, and to lead in jobs created by burgeoning industries of the future green economy.
Suggestions for how to incorporate climate change examples into math instruction will be noted in green text throughout the units.

Math Curriculum

Grade Eight

This eighth grade math curriculum aligns with the 2023 New Jersey Student Learning Standards (NJSLS). Instruction should be flexible and adaptable to meet the individual needs and learning styles of the students.

Instruction in grade eight focuses broadly on three critical areas:

- formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations;
- grasping the concept of a function and using functions to describe quantitative relationships;
- analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.

Domains and Cluster Areas

- **The Number System**
 - Know that there are numbers that are not rational and approximate them by rational numbers, and understand properties of rational and irrational numbers.
- **Expressions and Equations**

- Work with radicals and integer exponents.
- Understand the connections between proportional relationships, lines, and linear equations.
- Analyze and solve linear equations and pairs of simultaneous linear equations.
- **Functions**
 - Define, evaluate, and compare functions.
 - Use functions to model relationships between quantities.
- **Geometry**
 - Understand congruence and similarity using physical models, transparencies, or geometry software.
 - Understand and apply the Pythagorean Theorem.
 - Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.
- **Statistics and Probability**
 - Investigate patterns of association in bivariate data.

Emphases are given at the cluster level. Refer to the New Jersey Student Learning Standards for Mathematics for the specific standards that fall within each cluster. Even though the clusters have different levels of emphasis, this does not mean that standards can be omitted. All standards must be included in instruction throughout a grade level to ensure that students will be prepared for instruction at higher grade levels.

Indicator	Type	Cluster Heading
8.NS.A	S	Know that there are numbers that are not rational, and approximate them by rational numbers

8.EE.A	M	Work with radicals and integer exponents
8.EE.B	M	Understand the connections between proportional relationships, lines, and linear equations
8.EE.C	M	Analyze and solve linear equations and pairs of simultaneous linear equations
8.F.A	M	Define, evaluate and compare functions
8.F.B	M	Use functions to model relationships between quantities
8.G.A	M	Understand congruence and similarity using physical models, transparencies, or geometry software
8.G.B	M	Understand and apply the Pythagorean Theorem
8.G.C	A	Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.
8.SP.A	S	Investigate patterns of association in bivariate data

Key: M — Major, S — Supporting Clusters, A — Additional Clusters

(NJDOE, 2024)

Grade 8 Units and Pacing	
Unit 1: Rigid Transformations and Congruence	4 weeks

Unit 2: Dilations, Similarities, and Introducing Slope	4 weeks
Unit 3: Linear Relationships	4 weeks
Unit 4: Linear Equations and Linear Systems	5 weeks
Unit 5: Functions and Volume	5 weeks
Unit 6: Associations in Data	3 weeks
Unit 7: Exponents and Scientific Notation	4 weeks
Unit 8: Pythagorean Theorem and Irrational Numbers	4 weeks
Unit 9: Integrating Math Skills	2 weeks
Total Number of Weeks:	35 weeks

Unit 1: Rigid Transformations and Congruence

Unit Goal: In this unit, students learn to understand and use the terms “reflection,” “rotation,” “translation,” recognizing what determines each type of transformation, e.g., two points determine a translation. They learn to understand and use the terms “transformation” and “rigid transformation.” They identify and describe translations, rotations, and reflections, and sequences of these, using the terms “corresponding sides” and “corresponding angles,” and recognizing that lengths and angle measures are preserved. They draw images of figures under rigid transformations on and off square grids and the coordinate plane. They use rigid transformations to generate shapes and to reason about measurements of figures. They learn to understand congruence of plane figures in terms of rigid transformations. They recognize when one plane figure is congruent or not congruent to another. Students use the definition of “congruent” and properties of congruent figures to justify claims of congruence or non-congruence.

Duration: 4 weeks

Essential Questions:

- Transformations such as reflection, rotation, and translation are fundamental concepts in geometry, each with distinct characteristics and properties.
- A rigid transformation preserves both the size and shape of a figure, maintaining congruence.
- Corresponding sides and angles of figures under rigid transformations exhibit consistent relationships, demonstrating preservation of length and angle measures.
- The use of grids and coordinate planes facilitates visualization and analysis of rigid transformations.
- Congruence between plane figures can be determined through the application of rigid transformations, providing a rigorous basis for comparing geometric shapes.

Enduring Understandings:

- What distinguishes reflection, rotation, and translation as different types of geometric transformations?
- How does a rigid transformation preserve the properties of a geometric figure, and why is this concept important in geometry?
- In what ways do corresponding sides and angles demonstrate the preservation of length and angle measures under rigid transformations?
- How do square grids and coordinate planes aid in the representation and manipulation of figures undergoing rigid transformations?
- How can the concept of congruence be understood through the lens of rigid transformations, and how does it contribute to geometric reasoning and problem-solving?

Key NJSLS Strands:

- **8.G.A** Understand congruence and similarity using physical models, transparencies, or geometry software.
 - **8.G.A.1** Verify experimentally the properties of rotations, reflections, and translations:
 - **8.G.A.1a** Lines are transformed to lines, and line segments to line segments of the same length.
 - **8.G.A.1b** Angles are transformed to angles of the same measure.
 - **8.G.A.1c** Parallel lines are transformed to parallel lines.
 - **8.G.A.2** Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
 - **8.G.A.3** Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

- **8.G.A.5** Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.
- **8.EE.B** Understand and apply the Pythagorean Theorem
 - **8.EE.B.6** Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Interdisciplinary Connections

- **NJSLSA.R7** Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words.
- **NJSLSA.SL1** Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively.
- **NJSLSA.SL4** Present information, findings, and supporting evidence such that listeners can follow the line of reasoning and the organization, development, and style are appropriate to task, purpose, and audience.
- **MS-ETS1-1** Students estimate sums and differences while defining the criteria and constraints of a design problem with sufficient precision to ensure a successful solution, taking into account relevant scientific principles

21st-Century Skills

- **9.1.8.CR.2** Compare various ways to give back through strengths, passions, goals, and other personal factors.

Career Ready Practices

- **CRP2** Apply appropriate academic and technical skills.
- **CRP8** Utilize critical thinking to make sense of problems and persevere in solving them.
- **CRP11** Use technology to enhance productivity.

Technology Integration

- **9.4.8.TL.3** Select appropriate tools to organize and present information digitally.
- **9.4.8.TL.6** Collaborate to develop and publish work that provides perspectives on a real-world problem.

Skills	Instructional Topics	Assessment	Resources	Vocabulary
Verify that when a reflection, rotation, and/or translation is performed, lines are transformed to lines, and line segments to line segments of the same length.	Rigid Transformations Moving in the Plane Naming the Moves Grid Moves Making the Moves	Formative Assessments: Diagnostic Assessment Mid-topic Checkpoint Lesson Quizzes Lesson Self-assessment	Materials: blank paper colored pencils compasses copies of blackline master	vertex plane measure direction slide
Verify that when a reflection, rotation, and/or translation is performed, angles are transformed to angles of the same measure.	Coordinate Moves Describing Transformations Properties of Rigid Transformations	Teacher Observations Class Discussions Classwork/ Homework Exit Slips/Google Form	dried linguine pasta Four-function calculators	turn clockwise counterclockwise
Verify that when a reflection, rotation, and/or translation is performed, parallel lines are transformed to parallel lines	No Bending or Stretching Rotation Patterns Moves in Parallel	Summative Assessments: Topic/Unit Assessments Constructed Response Questions	tracing paper graph paper colored pencils scissors	reflection rotation translation opposite image
Know that two figures are congruent if one can be obtained from the other by a sequence of rotations, reflections, and/or translations.	Composing Figures Congruence What Is the Same? Congruent Polygons	Projects Cumulative Tests Performance Assessments	index cards graduated cylinders graph paper isometric graph paper	angle of rotation center (of rotation) line of reflection vertex
Describe a sequence of transformations that	Angles in a Triangle			sequence of

maps one congruent figure onto another Dilate, translate, rotate, and reflect two-dimensional figures on a coordinate plane. Describe the effects of dilations, translations, rotations, and reflections using coordinates Explain the sum of the interior angles of a triangle is 180 degrees. The measure of an exterior angle of a triangle is equal to the sum of the two remote interior angles. When parallel lines are cut by a transversal, corresponding, alternate interior, and alternate exterior angles are congruent. If two sets of corresponding angles in two triangles are congruent, then the	Alternate Interior Angles Adding the Angles in a Triangle Parallel Lines and the Angles in a Triangle Rotate and Tessellate	Alternate and Benchmark Assessments: Progress Monitoring Assessments Cumulative/Benchmark Assessments	long straightedge measuring tapes protractors rulers rulers marked with centimeters rulers marked with inches scientific calculators scissors spherical objects stopwatches straightedges string tape tools for creating a visual display toothpicks, pencils, straws, or other objects	transformations distance clockwise counterclockwise reflect rotate translate coordinate plane point segment coordinates x-axis y-axis polygon angle of rotation center (of rotation) line of reflection rigid transformation corresponding
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triangles are similar. Use facts about angles to construct an informal argument. Explain the Pythagorean Theorem states that the square of the hypotenuse of a Right triangle is equal to the sum of the squares of the other two sides. Explain a proof of the Pythagorean Theorem. Explain a proof of the converse of the Pythagorean Theorem.			tracing paper Activities: Moving Shapes Exploration Transformation Identification Game Grid Transformation Challenge Hands-On Shape Manipulation Coordinate Plane Transformations Task Descriptive Transformation Drawings No Bending Allowed! Exploring Rotation Patterns Parallel Transformation Sorting Shape Composition Puzzle Spot the Similarities	measurements preserve reflection rotation translation measure point midpoint segment vertical angles parallel intersect distance image rigid transformation midpoint parallel congruent perimeter
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			Congruent Polygon Matching Triangle Angle Investigation Parallel Lines and Triangle Angles Analysis Rotate and Tessellate Challenge Mentor Texts: <i>The Man Who Knew Infinity A Life of the Genius Ramanujan</i> by Robert Kanigel <i>Tessellations: The History and Making of Symmetrical Designs</i> by Eric Broug	area right angle -axis -axis area corresponding alternate interior angles transversal vertical angles congruent supplementary angles straight angle alternate interior angles transversal straight angle tessellation symmetry
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Modifications and Accommodations

Tier II and III Strategies	Small group instruction (Tier II) ● Explicit instruction ● Schema-based instruction ● Use visuals ● Use manipulatives ● Use a multi-sensory approach ● Guided notes and/or scaffold outline for written assignments ● Introduce key vocabulary before lesson ● Lesson taught again using a differentiated approach Small group or Individual instruction (Tier III) ● Intensive instruction ● Practice fact retrieval ● Lesson taught again using a differentiated approach ● Allow answers to be given orally or dictated ● Use visuals / Anchor Charts
Students with IEPs and 504 Plans	<u>Universal Design for Learning and Access for Students with Disabilities</u> <u>Movematics (dyscalculia.org)</u> Follow all the IEP/504 plan modifications Allow extra time to complete assignments or tests Guided notes and/or scaffold outline for written assignments Work in a small group Solidify and refine concepts through repetition Allow answers to be given orally or dictated

	Use multi-sensory teaching approaches Utilize assistive technology and materials Use large print books, Braille, or books on CD (digital text) Text to speech/audio recorded selections Amplification system as needed Leveled texts according to ability Fine motor skill stations embedded in rotation as needed Modified or constrained spelling word lists Provide anchor charts with high frequency words and phonemic patterns Students might need guidance as to what actions happen during partner work time. Invite students to discuss what it looks like when they are working with their partner while exploring the connecting cubes. Generate a list of shared expectations for partner work. Invite students to plan a strategy before they begin working. If time allows, invite students to share their plan with a partner. Some students may benefit from feedback that emphasizes effort and time on task. For example, moving counters in the squares of the 5-frame or filling the 5-frame with one counter in each square.
Multi-language Learners	<u>Mathematical Language Development and Access for English Learners</u> Work toward longer passages as skills in English increase Use visuals

	Introduce/highlight key vocabulary before lesson Teacher models reading aloud daily Provide peer tutoring Utilize translation supports including bilingual dictionary, translation device, assistive technology, and peers Guided notes and/or scaffold outline for written assignments Provide students with English Learner leveled readers Support recommended by the grade specific “can do” list outlined by WIDA, including Key Uses Edition and Descriptor Name Charts Create a visual display of what students want to do or make with the connecting cubes. As students share ideas, include a drawing or annotation to illustrate connections. Invite students to use gestures as they count.
Accelerating Students	<u>Accelerating Students Guide</u> Create an enhanced set of introductory activities (e.g. advance organizers, concept maps, concept puzzles) Provide options, alternatives and choices to differentiate and broaden the curriculum Organize and offer flexible small group learning activities Provide whole group enrichment explorations Teach cognitive and methodological skills

	Use center, stations, or contracts Organize integrated problem-solving simulations Propose interest-based extension activities Expose students to beyond level texts.
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(NJDOE, 2024)

Unit 2: Dilations, Similarities, and Introducing Slope

Unit Goal: In this unit, students learn to understand and use the term “dilation,” and to recognize that a dilation is determined by a point called the “center” and a number called the “scale factor.” They learn that under a dilation, the image of a circle is a circle and the image of a line is a line parallel to the original. They draw images of figures under dilations on and off the coordinate plane. They use the terms “corresponding sides” and “corresponding angles” to describe correspondences between a figure and its dilated image, and recognize that angle measures are preserved, but lengths are multiplied by the scale factor. They learn to understand similarity of plane figures in terms of rigid transformations and dilations. They learn to recognize when one plane figure is similar or not similar to another. They use the definition of “similar” and properties of similar figures to justify claims of similarity or non-similarity. Students learn the terms “slope” and “slope triangle,” and use the similarity of slope triangles on the same line to understand that any two distinct points on a line determine the same slope.

Duration: 4 weeks

Essential Questions:

- How is a dilation defined, and what are the key components that determine its properties?
- What geometric properties are preserved under a dilation, and how do these properties impact the relationships between figures and their dilated images?
- How does the concept of similarity extend our understanding of geometric transformations beyond congruence?
- What criteria are used to determine whether two plane figures are similar or not similar, and how are these criteria justified using properties of similar figures?
- How does the slope triangle concept contribute to our understanding of slope, and how is it related to the similarity of figures?
- In what ways do dilations and similarity play a role in real-world applications and problem-solving scenarios?

Enduring Understandings:

- Dilations are geometric transformations determined by a center and a scale factor, preserving the shape of figures while altering their size.
- Under a dilation, circles transform into circles, and lines transform into parallel lines.
- Corresponding sides and angles maintain relationships between a figure and its dilated image, with angle measures preserved and lengths multiplied by the scale factor.
- Similarity of plane figures can be understood through a combination of rigid transformations and dilations.
- The concept of similarity enables recognition of proportional relationships between figures, aiding in geometric reasoning and problem-solving.
- Slope triangles on the same line are similar, providing a geometric basis for understanding slope as a measure of steepness.

Key NJSLS Strands:

- **8.G.A** Understand congruence and similarity using physical models, transparencies, or geometry software
 - **8.G.A.2** Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
 - **8.G.A.3** Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
 - **8.G.A.4** Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
 - **8.G.A.5** Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.
- **8.EE.B** Understand and apply the Pythagorean Theorem
 - **8.EE.B.6** Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Interdisciplinary Connections

- **NJSLSA.R7** Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words.

● NJSLSA.SL1 Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively. ● NJSLSA.SL4 Present information, findings, and supporting evidence such that listeners can follow the line of reasoning and the organization, development, and style are appropriate to task, purpose, and audience. ● MS-PS3-3 Apply scientific principles to design, construct, and test a device that either minimizes or maximizes thermal energy transfer.
21st-Century Skills
● 9.2.8.CAP.3 Explain how career choices, educational choices, skills, economic conditions, and personal behavior affect income.
Career Ready Practices
● CRP2 Apply appropriate academic and technical skills. ● CRP8 Utilize critical thinking to make sense of problems and persevere in solving them. ● CRP11 Use technology to enhance productivity.
Technology Integration
● 9.4.8.TL.3 Select appropriate tools to organize and present information digitally. ● 9.4.8.TL.6 Collaborate to develop and publish work that provides perspectives on a real-world problem.

Skills	Instructional Topics	Assessment	Resources	Vocabulary
Explain two figures are congruent if one can be obtained from the other by a sequence of	Dilations Projecting and Scaling	Formative Assessments: Diagnostic Assessment	Materials: blank paper	scale factor scaled copy

rotations, reflections, and/or translations. Describe a sequence of transformations that maps one congruent figure onto another. Dilate, translate, rotate, and reflect two-dimensional figures on a coordinate plane. Describe the effects of dilations, translations, rotations, and reflections using coordinates. Explain two figures are similar if one can be obtained from the other by a sequence of dilations and rotations, reflections, and/or translations. Describe a sequence of transformations that maps one similar figure onto another. Know that the sum of the interior angles of a	Circular Grid Dilations with no Grid Dilations on a Square Grid More Dilations Similarity Similar Polygons Similar Triangles Side Length Quotients in Similar Triangles Slope Meet Slope Writing Equations for Lines Using Equations for Lines	Mid-topic Checkpoint Lesson Quizzes Lesson Self-assessment Teacher Observations Class Discussions Classwork/ Homework Exit Slips/Google Form Summative Assessments: Topic/Unit Assessments Constructed Response Questions Projects Cumulative Tests Performance Assessments Alternate and Benchmark Assessments: Progress Monitoring Assessments	colored pencils compasses copies of blackline master dried linguine pasta Four-function calculators tracing paper graph paper colored pencils scissors index cards graduated cylinders graph paper isometric graph paper long straightedge measuring tapes protractors	scaling dilation center of a dilation dilate center of a dilation scale factor similar dilate dilation quotient similar slope slope triangle similarity x-coordinate y-coordinate equation of a line quotient estimate
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triangle is 180 degrees. Explain that the measure of an exterior angle of a triangle is equal to the sum of the two remote interior angles. Know when parallel lines are cut by a transversal, corresponding, alternate interior, and alternate exterior angles are congruent. Understand if two sets of corresponding angles in two triangles are congruent, then the triangles are similar. Use facts about angles to construct an informal argument. Explain why the slope is the same between any two distinct points on a non-vertical line by drawing similar right triangles and comparing		Cumulative/Benchmark Assessments	rulers rulers marked with centimeters rulers marked with inches scientific calculators scissors spherical objects stopwatches straightedges string tape tools for creating a visual display toothpicks, pencils, straws, or other objects tracing paper Activities:	approximate / approximately
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the ratios of their sides. Drive the equation $y = mx$ for a line through the origin. Derive the equation $y = mx + b$ for a line intercepting the y-axis at b.			Dilation Discovery Circle and Line Dilation Exploration Corresponding Sides and Angles Investigation Similarity Assessment Real-world Similarity Scenarios Slope Triangle Analysis Proportional Relationships in Geometry Problem-solving with Similarity Mentor Texts: <i>Math Art: Truth, Beauty, and Equations</i> by Stephen Ornes	
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Modifications and Accommodations	
Tier II and III Strategies	Small group instruction (Tier II)

	● Explicit instruction ● Schema-based instruction ● Use visuals ● Use manipulatives ● Use a multi-sensory approach ● Guided notes and/or scaffold outline for written assignments ● Introduce key vocabulary before lesson ● Lesson taught again using a differentiated approach Small group or Individual instruction (Tier III) ● Intensive instruction ● Practice fact retrieval ● Lesson taught again using a differentiated approach ● Allow answers to be given orally or dictated ● Use visuals / Anchor Charts
Students with IEPs and 504 Plans	<u>Universal Design for Learning and Access for Students with Disabilities</u> <u>Movematics (dyscalculia.org)</u> Follow all the IEP/504 plan modifications Allow extra time to complete assignments or tests Guided notes and/or scaffold outline for written assignments Work in a small group Solidify and refine concepts through repetition Allow answers to be given orally or dictated Use multi-sensory teaching approaches

	Utilize assistive technology and materials Use large print books, Braille, or books on CD (digital text) Text to speech/audio recorded selections Amplification system as needed Leveled texts according to ability Fine motor skill stations embedded in rotation as needed Modified or constrained spelling word lists Provide anchor charts with high frequency words and phonemic patterns Students might need guidance as to what actions happen during partner work time. Invite students to discuss what it looks like when they are working with their partner while exploring the connecting cubes. Generate a list of shared expectations for partner work. Invite students to plan a strategy before they begin working. If time allows, invite students to share their plan with a partner. Some students may benefit from feedback that emphasizes effort and time on task. For example, moving counters in the squares of the 5-frame or filling the 5-frame with one counter in each square.
Multi-language Learners	<u>Mathematical Language Development and Access for English Learners</u> Work toward longer passages as skills in English increase Use visuals Introduce/highlight key vocabulary before lesson

	Teacher models reading aloud daily Provide peer tutoring Utilize translation supports including bilingual dictionary, translation device, assistive technology, and peers Guided notes and/or scaffold outline for written assignments Provide students with English Learner leveled readers Support recommended by the grade specific “can do” list outlined by WIDA, including Key Uses Edition and Descriptor Name Charts Create a visual display of what students want to do or make with the connecting cubes. As students share ideas, include a drawing or annotation to illustrate connections. Invite students to use gestures as they count.
Accelerating Students	<u>Accelerating Students Guide</u> Create an enhanced set of introductory activities (e.g. advance organizers, concept maps, concept puzzles) Provide options, alternatives and choices to differentiate and broaden the curriculum Organize and offer flexible small group learning activities Provide whole group enrichment explorations Teach cognitive and methodological skills Use center, stations, or contracts

	Organize integrated problem-solving simulations Propose interest-based extension activities Expose students to beyond level texts.
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(NJDOE, 2024)

Unit 3: Linear Relationships

Unit Goal: In this unit, students learn to understand and use the terms “rate of change,” “linear relationship,” and “vertical intercept.” They deepen their understanding of slope, and they learn to recognize connections among rate of change, slope, and constant of proportionality, and between linear and proportional relationships. They learn to understand that lines with the same slope are translations of each other. They represent linear relationships with tables, equations, and graphs that include lines with negative slopes or vertical intercepts, and horizontal and vertical lines. They learn to use the term “solution of an equation” when working with one or two linear equations in two variables, and learn to understand the graph of a linear equation as the set of its solutions. Students use these terms and representations in reasoning about situations involving one or two constant rates.

Duration: 4 weeks

Essential Questions:

- How does rate of change help us understand the relationship between two variables, and how is it calculated?
- What defines a linear relationship, and how does it differ from other types of relationships?
- What is the significance of slope in a linear relationship, and how does it relate to the rate of change and constant of proportionality?
- Why are lines with the same slope considered translations of each other, and what does this reveal about their relationships?
- How can linear relationships be represented using tables, equations, and graphs, and what insights do these representations offer?
- What does it mean for a point to be a solution of a linear equation, and how does this relate to its graphical representation?
- How does the graph of a linear equation represent the relationship between its variables, and what information does it convey about the equation?

Enduring Understandings:

- The concept of rate of change quantifies the relationship between two variables, illustrating how one variable changes with

respect to another.

- Linear relationships exhibit a constant rate of change, resulting in a straight-line graph.
- The slope of a line represents both the rate of change and the constant of proportionality in a linear relationship.
- Lines with the same slope are translations of each other, indicating equivalent rates of change.
- Linear relationships can be represented algebraically through equations, graphically with lines, and numerically with tables.
- Solutions of linear equations in two variables represent points that satisfy the equation and are graphically represented as points on the line.
- The graph of a linear equation represents the set of all its solutions, illustrating the relationship between variables.

Key NJSLS Strands:

- **8.EE.B** Understand the connections between proportional relationships, lines, and linear equations
 - **8.EE.B.5** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.
 - **8.EE.B.6** Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b .
- **8.EE.C** Analyze and solve linear equations and pairs of simultaneous linear equations.
 - **8.EE.C.8** Analyze and solve pairs of simultaneous linear equations.
 - **8.EE.C.8.a** Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
- **8.G.A** Understand congruence and similarity using physical models, transparencies, or geometry software
 - **8.G.A.1** Verify experimentally the properties of rotations, reflections, and translations.

Interdisciplinary Connections

- **NJSLSA.R7** Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words.

● NJSLSA.SL1 Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively. ● NJSLSA.SL4 Present information, findings, and supporting evidence such that listeners can follow the line of reasoning and the organization, development, and style are appropriate to task, purpose, and audience.
21st-Century Skills
● 9.1.8.PB.2 Explain how different circumstances can affect one's personal budget. ● 9.1.8.PB.4 Construct a simple personal savings and spending plan based on various sources of income and different stages of life (e.g. teenager, young adult, family). ● 9.1.8.PB.6 Construct a budget to save for short-term, long-term, and charitable goals.
Career Ready Practices
● CRP2 Apply appropriate academic and technical skills. ● CRP8 Utilize critical thinking to make sense of problems and persevere in solving them. ● CRP11 Use technology to enhance productivity.
Technology Integration
● 9.4.5.CT.3 Describe how digital tools and technology may be used to solve problems.

Skills	Instructional Topics	Assessment	Resources	Vocabulary
graph proportional relationships represented in different ways (i.e. ordered pairs, table, equation, phrases, etc.) recognize that for	Proportional Relationships Understanding Proportional Relationships	Formative Assessments: Diagnostic Assessment Mid-topic Checkpoint Lesson Quizzes	Materials: blank paper colored pencils compasses copies of blackline master	represent scale label constant of proportionality

proportional relationships, the unit rate is the slope of the graph compare the unit rates of two proportional relationships represented in different ways explain why the slope is the same between any two distinct points on a non-vertical line by drawing similar right triangles and comparing the ratios of their sides derive the equation $y = mx$ for a line through the origin derive the equation $y = mx + b$ for a line intercepting the y-axis at b solve solutions to a system of two linear equations in two variables correspond to points of intersection of	Graphs of Proportional Relationships Representing Proportional Relationships Comparing Proportional Relationships Representing Linear Relationships Introduction to Linear Relationships More Linear Relationships Representations of Linear Relationships Translating to Finding Slopes Slopes Don't Have to be Positive Calculating Slope Equations of All Kinds of Lines Linear Equations Solutions to Linear	Lesson Self-assessment Teacher Observations Class Discussions Classwork/ Homework Exit Slips/Google Form Summative Assessments: Topic/Unit Assessments Constructed Response Questions Projects Cumulative Tests Performance Assessments Alternate and Benchmark Assessments: Progress Monitoring Assessments Cumulative/Benchmark Assessments	dried linguine pasta Four-function calculators tracing paper graph paper colored pencils scissors index cards graduated cylinders graph paper isometric graph paper long straightedge measuring tapes protractors rulers rulers marked with centimeters rulers marked with inches scientific calculators	equation rate of change equation linear relationship constant rate slope vertical intercept -intercept initial (value or amount) constant rate relate horizontal intercept -intercept intersection point rate of change vertical intercept -intercept constraint horizontal line
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their graphs points of intersection satisfy both equations simultaneously solve systems of two linear equations in two variables algebraically estimate solutions of two linear equations in two variables by graphing the equations determine the number of solutions a system of two linear equations will have based upon inspection solve a system of two linear equations modeling real-world and mathematical problems verify that when a reflection, rotation, and/or translation is performed, lines are transformed to lines, and line segments to line segments of the same length	Equations More Solutions to Linear Equations Using Linear Relations to Solve Problems		scissors spherical objects stopwatches straightedges string tape tools for creating a visual display toothpicks, pencils, straws, or other objects tracing paper Activities: Proportional Plotting Rate Comparison Analysis Line Slope Exploration Equation Derivation Workshop Linear Equation Solution Quest	vertical line solution to an equation with two variables variable combination set of solutions
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verify that when a reflection, rotation, and/or translation is performed, angles are transformed to angles of the same measure verify that when a reflection, rotation, and/or translation is performed, parallel lines are transformed to parallel lines			Graphical Solution Estimation Solution Counting Challenge Real-world Systems Scenario Solving Line Transformation Verification Mentor Texts: <i>Math-terpieces: The Art of Problem-Solving</i> by Greg Tang <i>The Numberlys</i> by William Joyce and Christina Ellis	
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Modifications and Accommodations	
Tier II and III Strategies	Small group instruction (Tier II) • Explicit instruction • Schema-based instruction • Use visuals • Use manipulatives • Use a multi-sensory approach • Guided notes and/or scaffold outline for written assignments • Introduce key vocabulary before lesson • Lesson taught again using a differentiated approach

	Small group or Individual instruction (Tier III) ● Intensive instruction ● Practice fact retrieval ● Lesson taught again using a differentiated approach ● Allow answers to be given orally or dictated ● Use visuals / Anchor Charts
Students with IEPs and 504 Plans	<u>Universal Design for Learning and Access for Students with Disabilities</u> <u>Movematics (dyscalculia.org)</u> Follow all the IEP/504 plan modifications Allow extra time to complete assignments or tests Guided notes and/or scaffold outline for written assignments Work in a small group Solidify and refine concepts through repetition Allow answers to be given orally or dictated Use multi-sensory teaching approaches Utilize assistive technology and materials Use large print books, Braille, or books on CD (digital text) Text to speech/audio recorded selections Amplification system as needed Leveled texts according to ability Fine motor skill stations embedded in rotation as needed

	Modified or constrained spelling word lists Provide anchor charts with high frequency words and phonemic patterns Students might need guidance as to what actions happen during partner work time. Invite students to discuss what it looks like when they are working with their partner while exploring the connecting cubes. Generate a list of shared expectations for partner work. Invite students to plan a strategy before they begin working. If time allows, invite students to share their plan with a partner. Some students may benefit from feedback that emphasizes effort and time on task. For example, moving counters in the squares of the 5-frame or filling the 5-frame with one counter in each square.
Multi-language Learners	<u>Mathematical Language Development and Access for English Learners</u> Work toward longer passages as skills in English increase Use visuals Introduce/highlight key vocabulary before lesson Teacher models reading aloud daily Provide peer tutoring Utilize translation supports including bilingual dictionary, translation device, assistive technology, and peers Guided notes and/or scaffold outline for written assignments Provide students with English Learner leveled readers Support recommended by the grade specific “can do” list outlined by WIDA, including

	Key Uses Edition and Descriptor Name Charts Create a visual display of what students want to do or make with the connecting cubes. As students share ideas, include a drawing or annotation to illustrate connections. Invite students to use gestures as they count.
Accelerating Students	<u>Accelerating Students Guide</u> Create an enhanced set of introductory activities (e.g. advance organizers, concept maps, concept puzzles) Provide options, alternatives and choices to differentiate and broaden the curriculum Organize and offer flexible small group learning activities Provide whole group enrichment explorations Teach cognitive and methodological skills Use center, stations, or contracts Organize integrated problem-solving simulations Propose interest-based extension activities Expose students to beyond level texts.

(NJDOE, 2024)

Unit 4: Linear Equations and Linear Systems

Unit Goal: In this unit, students write and solve linear equations in one variable. These include equations in which the variable occurs on both sides of the equal sign, and equations with no solutions, exactly one solution, and infinitely many solutions. They learn that any one such equation is false, true for one value of the variable, or true for all values of the variable. They interpret solutions in the contexts from which the equations arose. Students write and solve systems of linear equations in two variables and interpret the solutions in the contexts from which the equations arose. They learn what is meant by a solution for a system of equations, namely that a solution of the system is a solution for each equation in the system. Students use the understanding that each pair of values that make an equation true are coordinates of a point on the graph of the equation and conversely that the coordinates of each point on the graph of an equation make the equation true. Thus, a pair of values that satisfies a system of equations are coordinates of a point that lies on the graphs of all the equations in the system, and, conversely, a point that lies on the graphs of all the equations in the system has coordinates that satisfy all the equations in the system. Students learn to understand and use the terms “system of equations,” “solution for the system of equations,” “zero solutions,” “no solution,” “one solution,” and “infinitely many solutions.”

Duration: 5 weeks

Essential Questions:

- How do we determine the number of solutions to a linear equation in one variable, and what does each type of solution represent?
- What does it mean for a linear equation to be true, false, or true for all values of the variable, and how does this relate to its solution(s)?
- How can we interpret solutions to linear equations within the context of the problems they represent?
- What is a system of linear equations, and how does it relate to the relationships between multiple variables or quantities?
- How do we determine the solutions for a system of linear equations, and what do these solutions represent graphically and contextually?
- Why do solutions to linear equations correspond to points on their graphs, and how does this relationship help us solve equations and interpret solutions?
- How do the terms related to systems of equations help us describe and analyze the properties and outcomes of solving linear systems?

Enduring Understandings:

- Linear equations in one variable can have zero solutions, exactly one solution, or infinitely many solutions, depending on the relationship between the variables.
- The truth of a linear equation depends on the value(s) of the variable(s) involved, with each solution satisfying the equation.

- Solutions to linear equations have real-world interpretations relevant to the contexts from which the equations arose.
- Systems of linear equations in two variables represent multiple relationships between quantities, and their solutions correspond to points of intersection on the graphs of the equations.
- A solution for a system of equations is a set of values that satisfies all equations within the system, indicating a common point of intersection on their graphs.
- Every pair of values that satisfies an equation corresponds to a point on the graph of the equation, and conversely, every point on the graph satisfies the equation.
- The terms "system of equations," "solution for the system of equations," "zero solutions," "no solution," "one solution," and "infinitely many solutions" are used to describe the properties and outcomes of solving linear systems.

Key NJSLS Strands:

- **8.EE.B** Understand the connections between proportional relationships, lines, and linear equations
 - **8.EE.B.5** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.
 - **8.EE.B.6** Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b .
- **8.EE.C** Analyze and solve linear equations and pairs of simultaneous linear equations.
 - **8.EE.C.7** Solve linear equations in one variable.
 - **8.EE.C.7.a** Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).
 - **8.EE.C.7.b** Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.
 - **8.EE.C.8** Analyze and solve pairs of simultaneous linear equations.
 - **8.EE.C.8.a** Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
 - **8.EE.C.8.b** Solve systems of two linear equations in two variables using the substitution method and estimate solutions by graphing the equations. Solve simple cases by inspection.
 - **8.EE.C.8.c** Solve real-world and mathematical problems leading to two linear equations in two variables.

Interdisciplinary Connections				
● NJSLSA.R7 Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words. ● NJSLSA.SL1 Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively. ● NJSLSA.SL4 Present information, findings, and supporting evidence such that listeners can follow the line of reasoning and the organization, development, and style are appropriate to task, purpose, and audience. ● MS-ETS1-1 Students estimate sums and differences while defining the criteria and constraints of a design problem with sufficient precision to ensure a successful solution, taking into account relevant scientific principles.				
21st-Century Skills				
● 9.4.8.CT.2 Develop multiple solutions to a problem and evaluate short- and long-term effects to determine the most plausible option (e.g., MS-ETS1-4, 6.1.8.CivicsDP.1). ● 9.4.8.IML.12 Use relevant tools to produce, publish, and deliver information supported with evidence for an authentic audience.				
Career Ready Practices				
● CRP2 Apply appropriate academic and technical skills.				
Technology Integration				
● 9.4.5.CT.3 Describe how digital tools and technology may be used to solve problems.				

Skills	Instructional Topics	Assessment	Resources	Vocabulary
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Graph proportional relationships represented in different ways (i.e. ordered pairs, table, equation, phrases, etc.) Recognize that for proportional relationships, the unit rate is the slope of the graph Compare the unit rates of two proportional relationships represented in different ways Explain why the slope is the same between any two distinct points on a non-vertical line by drawing similar right triangles and comparing the ratios of their sides Derive the equation $y = mx$ for a line through the origin Derive the equation $y = mx + b$ for a line intercepting the y-axis at	Number Puzzles Linear Equations in One Variable Keeping the Equation Balanced Balanced Moves More Balanced Moves Solving Any Linear Equation Strategic Solving All, Some, or No Solutions How Many Solutions? When Are They the Same? Systems of Linear Equations On or Off the Line? On Both of the Lines Solving Systems of Equations Solving More Systems	Formative Assessments: Diagnostic Assessment Mid-topic Checkpoint Lesson Quizzes Lesson Self-assessment Teacher Observations Class Discussions Classwork/ Homework Exit Slips/Google Form Summative Assessments: Topic/Unit Assessments Constructed Response Questions Projects Cumulative Tests Performance Assessments	Materials: blank paper colored pencils compasses copies of blackline master dried linguine pasta Four-function calculators tracing paper graph paper colored pencils scissors index cards graduated cylinders graph paper isometric graph paper long straightedge	representation expression solution to an equation distribute substitute equation term like terms distributive property factor term like terms distribute common denominator no solution (only) one solution constant term coefficient linear equation
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b Explain that a linear equation in one variable can result in one solution, infinitely many solutions, or no solution Show which of these outcomes is the case by transforming the original equation into the form $x = a$, $a = a$, or $a = b$ Solve linear equations in one variable with rational number coefficients, including equations that require expanding expressions using the distributive property and combining like terms Describe solutions to a system of two linear equations in two variables Correspond to points of intersection of their graphs	Writing Systems of Equations Solving Problems with Systems of Equations	Alternate and Benchmark Assessments: Diagnostic Assessment Progress Monitoring Assessments Cumulative/Benchmark Assessments	measuring tapes protractors rulers rulers marked with centimeters rulers marked with inches scientific calculators scissors spherical objects stopwatches straightedges string tape tools for creating a visual display toothpicks, pencils, straws, or other objects	infinitely many solutions expression variable ordered pair system of equations solution to a system of equations substitution substitute no solution (only) one solution infinitely many solutions algebraically system of equations substitution
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Explain that points of intersection satisfy both equations simultaneously Solve systems of two linear equations in two variables algebraically Estimate solutions of two linear equations in two variables by graphing the equations Determine the number of solutions a system of two linear equations will have based upon inspection Solve a system of two linear equations modeling real-world and mathematical problems			tracing paper Activities: Slope Explanation Challenge Linear Equation Results Discussion Rational Coefficients Equation Solving Task System Solution Description Exercise Algebraic System Solution Practice Graphical System Solution Estimation Activity System Solution Counting Task Proportional Relationship Graphing Challenge Unit Rate Slope Recognition Exercise	
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			Unit Rate Comparison Task Origin Line Equation Derivation Activity Intercept Line Equation Derivation Exercise Linear Equation Result Analysis Real-world Systems Problem Solving Task Mentor Texts: <i>Math Potatoes:</i> <i>Mind-stretching Brain Food</i> by Greg Tang	
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Modifications and Accommodations	
Tier II and III Strategies	Small group instruction (Tier II) ● Explicit instruction ● Schema-based instruction ● Use visuals ● Use manipulatives ● Use a multi-sensory approach ● Guided notes and/or scaffold outline for written assignments ● Introduce key vocabulary before lesson

	• Lesson taught again using a differentiated approach Small group or Individual instruction (Tier III) • Intensive instruction • Practice fact retrieval • Lesson taught again using a differentiated approach • Allow answers to be given orally or dictated • Use visuals / Anchor Charts
Students with IEPs and 504 Plans	<u>Universal Design for Learning and Access for Students with Disabilities</u> <u>Movematics (dyscalculia.org)</u> Follow all the IEP/504 plan modifications Allow extra time to complete assignments or tests Guided notes and/or scaffold outline for written assignments Work in a small group Solidify and refine concepts through repetition Allow answers to be given orally or dictated Use multi-sensory teaching approaches Utilize assistive technology and materials Use large print books, Braille, or books on CD (digital text) Text to speech/audio recorded selections Amplification system as needed Leveled texts according to ability

	Fine motor skill stations embedded in rotation as needed Modified or constrained spelling word lists Provide anchor charts with high frequency words and phonemic patterns Students might need guidance as to what actions happen during partner work time. Invite students to discuss what it looks like when they are working with their partner while exploring the connecting cubes. Generate a list of shared expectations for partner work. Invite students to plan a strategy before they begin working. If time allows, invite students to share their plan with a partner. Some students may benefit from feedback that emphasizes effort and time on task. For example, moving counters in the squares of the 5-frame or filling the 5-frame with one counter in each square.
Multi-language Learners	<u>Mathematical Language Development and Access for English Learners</u> Work toward longer passages as skills in English increase Use visuals Introduce/highlight key vocabulary before lesson Teacher models reading aloud daily Provide peer tutoring Utilize translation supports including bilingual dictionary, translation device, assistive technology, and peers Guided notes and/or scaffold outline for written assignments Provide students with English Learner leveled readers

	Support recommended by the grade specific “can do” list outlined by WIDA, including Key Uses Edition and Descriptor Name Charts Create a visual display of what students want to do or make with the connecting cubes. As students share ideas, include a drawing or annotation to illustrate connections. Invite students to use gestures as they count.
Accelerating Students	<u>Accelerating Students Guide</u> Create an enhanced set of introductory activities (e.g. advance organizers, concept maps, concept puzzles) Provide options, alternatives and choices to differentiate and broaden the curriculum Organize and offer flexible small group learning activities Provide whole group enrichment explorations Teach cognitive and methodological skills Use center, stations, or contracts Organize integrated problem-solving simulations Propose interest-based extension activities Expose students to beyond level texts.

(NJDOE, 2024)

Unit 5: Functions and Volume

Unit Goal: In this unit, students are introduced to the concept of a function. They learn to understand and use the terms “input,”

“output,” and “function,” e.g., “temperature is a function of time.” They describe functions as increasing or decreasing between specific numerical inputs, and they consider the inputs of a function to be values of its independent variable and its outputs to be values of its dependent variable.

Duration: 5 weeks

Essential Questions:

- What is a function, and how does it relate to the concepts of inputs and outputs?
- How do we determine the input and output variables in a given function, and why is this distinction important?
- What does it mean for a function to be increasing or decreasing, and how can we identify these characteristics?
- How do we analyze the behavior of functions between specific numerical inputs, and what insights does this provide?
- In what ways can functions be applied to model and interpret relationships between quantities in real-world situations?

Enduring Understandings:

- Functions relate inputs to outputs, where each input corresponds to exactly one output.
- The terms "input," "output," and "function" are used to describe relationships between variables, with the input representing the independent variable and the output representing the dependent variable.
- Functions can be described as increasing or decreasing based on changes in the input variable.
- The behavior of functions can be analyzed between specific numerical inputs to determine their trend.
- Understanding functions helps us model and interpret relationships between quantities in various contexts.

Key NJSLS Strands:

- **8.F.A** Define, evaluate and compare functions.
 - **8.F.A.1** Compare properties (e.g. rate of change, intercepts, domain and range) of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
 - **8.F.A.2** Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.
 - **8.F.A.3** Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.
- **8.F.B** Use functions to model relationships between quantities.
 - **8.F.B.4** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these

from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

- **8.F.B.5** Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.
- **8.G.C** Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres
 - **8.G.C.9** Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Interdisciplinary Connections

- **NJSLSA.R7** Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words.
- **NJSLSA.SL1** Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively.
- **NJSLSA.SL4** Present information, findings, and supporting evidence such that listeners can follow the line of reasoning and the organization, development, and style are appropriate to task, purpose, and audience.
- **MS-ETS1-1** Students estimate sums and differences while defining the criteria and constraints of a design problem with sufficient precision to ensure a successful solution, taking into account relevant scientific principles.

21st-Century Skills

- **9.2.8.CAP.3** Explain how career choices, educational choices, skills, economic conditions, and personal behavior affect income.

Career Ready Practices

- **CRP4** Communicate clearly and effectively with reason.

Technology Integration
● 9.4.8.TL.6 Collaborate to develop and publish work that provides perspectives on a real-world problem.

Skills	Instructional Topics	Assessment	Resources	Vocabulary
Explain a function is a rule that assigns to each input exactly one output Describe that the graph of a function is the set of ordered pairs consisting of an input and the corresponding output Describe qualitatively the functional relationships between two quantities by analyzing a graph Sketch a graph that exhibits the qualitative features of a function given a verbal description Explain the equation $y = mx + b$ defines a linear	Inputs and Outputs Introduction to Functions Representing and Interpreting Functions Equations for Functions Tables, Equations, and Graphs of Functions More Graphs of Functions Even More Graphs of Functions Connecting Representations of Functions Linear Functions and Rates of Change Linear Functions	Formative Assessments: Diagnostic Assessment Mid-topic Checkpoint Lesson Quizzes Lesson Self-assessment Teacher Observations Class Discussions Classwork/ Homework Exit Slips/Google Form Summative Assessments: Topic/Unit Assessments Constructed Response Questions Projects	Materials: blank paper colored pencils compasses copies of blackline master dried linguine pasta Four-function calculators tracing paper graph paper colored pencils scissors index cards	input output function depends on independent variable dependent variable radius prediction volume cube functional relationship linear function function mathematical model

function	Linear Models	Cumulative Tests	graduated cylinders	prediction
Interpret a set of points forming a straight line as the graph of a linear function	Piecewise Linear Functions	Performance Assessments	graph paper	piecewise linear function
Graph linear equations	Cylinders and Cones	Alternate and Benchmark Assessments:	isometric graph paper	constant rate
Give examples of nonlinear functions	Filling Containers	Progress Monitoring Assessments	long straightedge	cylinder
Construct a function to model a linear relationship between two quantities	How Much Will Fit?	Cumulative/Benchmark Assessments	measuring tapes	three-dimensional
Determine the rate of change and initial value of a function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph	The Volume of a Cylinder		protractors	cone
	Finding Cylinder Dimensions		rulers	sphere
	The Volume of a Cone		rulers marked with centimeters	dimension
	Finding Cone Dimensions		rulers marked with inches	cylinder
	Dimensions and Spheres		scientific calculators	cube
	Scaling One Dimension		scissors	cubic centimeter
	Scaling Two Dimensions		spherical objects	rectangular prism
Interpret the rate of change and initial value of a function in terms of the situation it models	Estimating a Hemisphere		stopwatches	base (of a cylinder or cone)
Graph proportional relationships represented in different	The Volume of a Sphere		straightedges	approximation for
	Cylinders, Cones, and Spheres		string	cone
			tape	hemisphere
				sphere
				spherical

ways (i.e. ordered pairs, table, equation, phrases, etc.) Recognize that for proportional relationships, the unit rate is the slope of the graph Compare the unit rates of two proportional relationships represented in different ways Apply the formulas for volume of a cone, cylinder, or sphere in a real world context Calculate the volume of a cone, cylinder, or sphere Find a missing dimension of a cone, cylinder or sphere given its volume	Volume As a Function of . .		tools for creating a visual display toothpicks, pencils, straws, or other objects tracing paper Activities: Linear Equation Exploration Graph Analysis Challenge Linear Equation Graphing Practice Nonlinear Function Sorting Real-World Linear Modeling Task Rate of Change Investigation Contextual Rate of Change Analysis	approximate range
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			Proportional Relationship Graphing Activity Slope as Unit Rate Exercise Unit Rate Comparison Task Real-World Volume Application Problems Calculation and Dimension Mentor Texts: <i>One Grain of Rice: A Mathematical Folktale</i> by Demi	
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Modifications and Accommodations	
Tier II and III Strategies	Small group instruction (Tier II) ● Explicit instruction ● Schema-based instruction ● Use visuals ● Use manipulatives ● Use a multi-sensory approach ● Guided notes and/or scaffold outline for written assignments ● Introduce key vocabulary before lesson

	• Lesson taught again using a differentiated approach Small group or Individual instruction (Tier III) • Intensive instruction • Practice fact retrieval • Lesson taught again using a differentiated approach • Allow answers to be given orally or dictated • Use visuals / Anchor Charts
Students with IEPs and 504 Plans	<u>Universal Design for Learning and Access for Students with Disabilities</u> <u>Movematics (dyscalculia.org)</u> Follow all the IEP/504 plan modifications Allow extra time to complete assignments or tests Guided notes and/or scaffold outline for written assignments Work in a small group Solidify and refine concepts through repetition Allow answers to be given orally or dictated Use multi-sensory teaching approaches Utilize assistive technology and materials Use large print books, Braille, or books on CD (digital text) Text to speech/audio recorded selections Amplification system as needed Leveled texts according to ability

	Fine motor skill stations embedded in rotation as needed Modified or constrained spelling word lists Provide anchor charts with high frequency words and phonemic patterns Students might need guidance as to what actions happen during partner work time. Invite students to discuss what it looks like when they are working with their partner while exploring the connecting cubes. Generate a list of shared expectations for partner work. Invite students to plan a strategy before they begin working. If time allows, invite students to share their plan with a partner. Some students may benefit from feedback that emphasizes effort and time on task. For example, moving counters in the squares of the 5-frame or filling the 5-frame with one counter in each square.
Multi-language Learners	<u>Mathematical Language Development and Access for English Learners</u> Work toward longer passages as skills in English increase Use visuals Introduce/highlight key vocabulary before lesson Teacher models reading aloud daily Provide peer tutoring Utilize translation supports including bilingual dictionary, translation device, assistive technology, and peers Guided notes and/or scaffold outline for written assignments Provide students with English Learner leveled readers

	Support recommended by the grade specific “can do” list outlined by WIDA, including Key Uses Edition and Descriptor Name Charts Create a visual display of what students want to do or make with the connecting cubes. As students share ideas, include a drawing or annotation to illustrate connections. Invite students to use gestures as they count.
Accelerating Students	<u>Accelerating Students Guide</u> Create an enhanced set of introductory activities (e.g. advance organizers, concept maps, concept puzzles) Provide options, alternatives and choices to differentiate and broaden the curriculum Organize and offer flexible small group learning activities Provide whole group enrichment explorations Teach cognitive and methodological skills Use center, stations, or contracts Organize integrated problem-solving simulations Propose interest-based extension activities Expose students to beyond level texts.

(NJDOE, 2024)

Unit 6: Associations in Data

Unit Goal: In this unit, students generate and work with bivariate data sets that has more variability than in previous units. They learn to understand and use the terms “scatter plot” and “association,” and describe associations as “positive” or “negative” and “linear” or “nonlinear.” Students describe scatter plots, using a term previously used to describe univariate data “cluster,” and the new term “outlier.” They fit lines to scatter plots and informally assess their goodness of fit by judging the closeness of the data points to the lines, and compare predicted and actual values. Students learn to understand and use the terms “two-way table,” “bar graph,” and “segmented bar graph,” using two-way tables to investigate categorical data.

Duration: 3 weeks

Essential Questions:

- How does bivariate data differ from univariate data, and why is it important to analyze relationships between two variables?
- What are scatter plots, and how do they help us understand the association between two variables?
- How do we categorize associations in scatter plots as positive or negative, and linear or non-linear? What visual cues do we look for?
- What role do clusters and outliers play in interpreting scatter plots, and how do they impact our understanding of the relationship between variables?
- How do we fit lines to scatter plots, and how do we determine the goodness of fit?
- What are two-way tables, and how are they used to organize and analyze categorical data?
- How do bar graphs and segmented bar graphs help us visualize and compare categorical data sets, and what insights do they provide into the data distribution and relationships?

Enduring Understandings:

- Bivariate data sets exhibit more variability than univariate data sets, allowing for the exploration of relationships between two variables.
- Scatter plots are visual representations of bivariate data that help analyze the relationship between two variables.
- Associations between variables in scatter plots can be categorized as positive or negative, and linear or non-linear, based on the direction and shape of the plotted points.
- Scatter plots may contain clusters of data points as well as outliers, which can influence the interpretation of the association between variables.
- Lines can be fitted to scatter plots to model the relationship between variables, and the goodness of fit can be informally assessed by comparing the data points to the line.
- Two-way tables are used to organize categorical data for analysis and comparison.
- Bar graphs and segmented bar graphs are graphical representations of categorical data, providing insights into the

distribution and relationships within the data.

Key NJSLS Strands:

- **8.SP.A** Investigate patterns of association in bivariate data.
 - **8.SP.A.1** Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
 - **8.SP.A.2** Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit (e.g. line of best fit) by judging the closeness of the data points to the line.
 - **8.SP.A.3** Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.
 - **8.SP.A.4** Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.

Interdisciplinary Connections

- **NJSLSA.R7** Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words.
- **NJSLSA.SL1** Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively
- **NJSLSA.SL4** Present information, findings, and supporting evidence such that listeners can follow the line of reasoning and the organization, development, and style are appropriate to task, purpose, and audience.
- **MS-ETS1-1** Students estimate sums and differences while defining the criteria and constraints of a design problem with sufficient precision to ensure a successful solution, taking into account relevant scientific principles.

21st-Century Skills

● 9.4.8.CT.2 Develop multiple solutions to a problem and evaluate short- and long-term effects to determine the most plausible option).
Career Ready Practices
● CRP2 Apply appropriate academic and technical skills.
Technology Integration
● 9.4.5.CT.3 Describe how digital tools and technology may be used to solve problems.

Skills	Instructional Topics	Assessment	Resources	Vocabulary
Construct scatter plots Interpret scatter plots to investigate patterns of association between two quantities Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association Explain that straight lines are used to model relationships between two quantitative	Does This Predict That? Organizing Data Plotting Data Associations in Numerical Data What a Point in a Scatter Plot Means Fitting a Line to Data Describing Trends in Scatter Plots The Slope of a Fitted	Formative Assessments: Diagnostic Assessment Mid-topic Checkpoint Lesson Quizzes Lesson Self-assessment Teacher Observations Class Discussions Classwork/ Homework Exit Slips/Google Form	Materials: blank paper colored pencils compasses copies of blackline master dried linguine pasta Four-function calculators tracing paper	scatter plot data display attribute numerical data categorical data outlier predict overpredict underpredict linear model

variables Informally fit a straight line for scatter plots that suggest a linear association Informally assess the fit of the line for a scatter plot by judging the closeness of the data points to the line Interpret the slope and intercept in the context of bivariate measurement data using the equation of a linear model Explain that two-way tables can be used to show patterns of association in categorical data Construct a two-way table summarizing data on two categorical variables collected from the same subjects Interpret a two-way table by identifying joint	Line Observing More Patterns in Scatter Plots Analyzing Bivariate Data Associations in Categorical Data Looking for Associations Using Data Displays to Find Associations	Summative Assessments: Topic/Unit Assessments Constructed Response Questions Projects Cumulative Tests Performance Assessments Alternate and Benchmark Assessments: Progress Monitoring Assessments Cumulative/Benchmark Assessments	graph paper colored pencils scissors index cards graduated cylinders graph paper isometric graph paper long straightedge measuring tapes protractors rulers rulers marked with centimeters rulers marked with inches scientific calculators scissors spherical objects	positive association negative association linear association nonlinear association no association fitted line cluster independent variable dependent variable positive association negative association linear association segmented bar graph relative frequency two-way (frequency) table scatter plot outlier cluster
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frequencies and calculating marginal frequencies Use relative frequencies calculated for rows or columns to describe possible association between the two variables			stopwatches straightedges string tape tools for creating a visual display toothpicks, pencils, straws, or other objects tracing paper Activities: Scatter Plot Construction Challenge Scatter Plot Interpretation Investigation Describing Patterns in Scatter Plots Exercise Modeling Relationships with Straight Lines Activity	
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			Informal Straight Line Fitting Task Assessing Line Fit Practice Interpreting Slope and Intercept Exploration Two-Way Table Usage Activity Two-Way Table Construction Task Interpreting Two-Way Tables Exercise Mentor Texts: <i>The Girl With a Mind For Math: The Story of Raye Montague</i> by Julia Finley Mosca <i>The Cartoon Guide to Statistics</i> by Larry Gonick	
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Modifications and Accommodations	
Tier II and III Strategies	Small group instruction (Tier II) • Explicit instruction

	● Schema-based instruction ● Use visuals ● Use manipulatives ● Use a multi-sensory approach ● Guided notes and/or scaffold outline for written assignments ● Introduce key vocabulary before lesson ● Lesson taught again using a differentiated approach Small group or Individual instruction (Tier III) ● Intensive instruction ● Practice fact retrieval ● Lesson taught again using a differentiated approach ● Allow answers to be given orally or dictated ● Use visuals / Anchor Charts
Students with IEPs and 504 Plans	<u>Universal Design for Learning and Access for Students with Disabilities</u> <u>Movematics (dyscalculia.org)</u> Follow all the IEP/504 plan modifications Allow extra time to complete assignments or tests Guided notes and/or scaffold outline for written assignments Work in a small group Solidify and refine concepts through repetition Allow answers to be given orally or dictated Use multi-sensory teaching approaches Utilize assistive technology and materials

	Use large print books, Braille, or books on CD (digital text) Text to speech/audio recorded selections Amplification system as needed Leveled texts according to ability Fine motor skill stations embedded in rotation as needed Modified or constrained spelling word lists Provide anchor charts with high frequency words and phonemic patterns Students might need guidance as to what actions happen during partner work time. Invite students to discuss what it looks like when they are working with their partner while exploring the connecting cubes. Generate a list of shared expectations for partner work. Invite students to plan a strategy before they begin working. If time allows, invite students to share their plan with a partner. Some students may benefit from feedback that emphasizes effort and time on task. For example, moving counters in the squares of the 5-frame or filling the 5-frame with one counter in each square.
Multi-language Learners	<u>Mathematical Language Development and Access for English Learners</u> Work toward longer passages as skills in English increase Use visuals Introduce/highlight key vocabulary before lesson Teacher models reading aloud daily

	Provide peer tutoring Utilize translation supports including bilingual dictionary, translation device, assistive technology, and peers Guided notes and/or scaffold outline for written assignments Provide students with English Learner leveled readers Support recommended by the grade specific “can do” list outlined by WIDA, including Key Uses Edition and Descriptor Name Charts Create a visual display of what students want to do or make with the connecting cubes. As students share ideas, include a drawing or annotation to illustrate connections. Invite students to use gestures as they count.
Accelerating Students	<u>Accelerating Students Guide</u> Create an enhanced set of introductory activities (e.g. advance organizers, concept maps, concept puzzles) Provide options, alternatives and choices to differentiate and broaden the curriculum Organize and offer flexible small group learning activities Provide whole group enrichment explorations Teach cognitive and methodological skills Use center, stations, or contracts Organize integrated problem-solving simulations

	Propose interest-based extension activities Expose students to beyond level texts.
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(NJDOE, 2024)

Unit 7: Exponents and Scientific Notation

Unit Goal: In this unit, students extend the definition of exponents to include all integers, and in the process codify the properties of exponents. They apply these concepts to the base-ten system, and learn about orders of magnitude and scientific notation in order to represent and compute with very large and very small quantities.

Duration: 4 weeks

Essential Questions:

- How can we extend the concept of exponents beyond whole numbers to include negative integers and fractions?
- What are the fundamental properties of exponents, and how do they simplify expressions involving powers?
- Why is scientific notation a valuable tool for representing extremely large and small quantities?
- How does understanding orders of magnitude help us grasp the scale and relative size of different quantities?
- In what ways does the base-ten system underpin our understanding of exponential notation and scaling in mathematics and science?
- How do we apply exponential concepts to real-world scenarios, such as in scientific research or financial calculations?

Enduring Understandings:

- Exponents can be extended to include all integers, enabling efficient representation and computation of large and small quantities.
- The properties of exponents provide systematic rules for simplifying and manipulating expressions involving powers.
- Understanding orders of magnitude and scientific notation facilitates the representation and computation of quantities spanning vast ranges.
- The base-ten system serves as a foundational framework for comprehending exponential notation and scaling in mathematics and science.

Key NJSLS Strands:

- **8.EE.A** Work with radicals and integer exponents

- **8.EE.A.1** Know and apply the properties of integer exponents to generate equivalent numerical expressions.
- **8.EE.A.3** Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.
- **8.EE.A.4** Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Interdisciplinary Connections

- **ELA: W.6.1** When students are justifying their reasoning on short answer and extended constructed response questions they write arguments to support claims with clear reasons and relevant evidence.
- **MS-ESS1-3** When students are creating and comparing ratios they analyze and interpret data to determine scale properties of objects in the solar system

21st-Century Skills

- **9.2.8.CAP.12** Assess personal strengths, talents, values, and interests to appropriate jobs and careers to maximize career potential.

Career Ready Practices

- **CRP2** Apply appropriate academic and technical skills.
- **CRP8** Utilize critical thinking to make sense of problems and persevere in solving them.
- **CRP11** Use technology to enhance productivity.

Technology Integration

- **9.4.8.TL.3** Select appropriate tools to organize and present information digitally.
- **9.4.5.CT.3** Describe how digital tools and technology may be used to solve problems.

Skills	Instructional Topics	Assessment	Resources	Vocabulary
Know the properties of integer exponents Determine whether two numerical expressions involving integer exponents are equivalent Generate equivalent expressions using the properties of exponents Estimate a very large or very small number as a single digit times an integer power of ten Express how many times larger one quantity is compared to another when written as a single digit times an integer power of ten Add, subtract, multiply, and divide numbers expressed in scientific notation Add, subtract, multiply,	Exponent Review Exponent Rules Multiplying Powers of Ten Powers of Powers of 10 Dividing Powers of 10 Negative Exponents with Powers of 10 What about Other Bases? Practice with Rational Bases Combining Bases Scientific Notation Describing Large and Small Numbers Using Powers of 10 Representing Large Numbers on the Number Line Representing Small	Formative Assessments: Diagnostic Assessment Mid-topic Checkpoint Lesson Quizzes Lesson Self-assessment Teacher Observations Class Discussions Classwork/ Homework Exit Slips/Google Form Summative Assessments: Topic/Unit Assessments Constructed Response Questions Projects Cumulative Tests Performance Assessments	Materials: blank paper colored pencils compasses copies of blackline master dried linguine pasta Four-function calculators tracing paper graph paper colored pencils scissors index cards graduated cylinders graph paper isometric graph paper	exponent power factor reciprocal repeated multiplication powers of 10 base (of an exponent) power of powers expanded positive exponent zero exponent negative exponent positive exponent exponent base (of an exponent) power zero exponent

and divide numbers where one is expressed in decimal notation and the other is expressed in scientific notation Choose appropriate units to represent measurements of very large or very small quantities Interpret scientific notation generated by technology as a number multiplied by a power of ten	Numbers on the Number Line Applications of Arithmetic with Powers of 10 Definition of Scientific Notation Multiplying, Dividing, and Estimating with Scientific Notation Adding and Subtracting with Scientific Notation	Alternate and Benchmark Assessments: Progress Monitoring Assessments Cumulative/Benchmark Assessments	long straightedge measuring tapes protractors rulers rulers marked with centimeters rulers marked with inches scientific calculators scissors spherical objects stopwatches straightedges string tape tools for creating a visual display	reciprocal evaluate factor power of powers negative exponent square (of a number) billion trillion multiple of integer multiple of scientific notation integer powers of 10 billion trillion scientific notation
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			toothpicks, pencils, straws, or other objects tracing paper Activities: Property Investigation Equivalence Exploration Expression Generation Estimation Exercise Comparative Analysis Scientific Notation Arithmetic Mixed Notation Operations Unit Selection Challenge Technology Integration Mentor Texts: <i>Math Curse</i> by Jon Scieszka and Lane Smith	
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Modifications and Accommodations	
Tier II and III Strategies	Small group instruction (Tier II) ● Explicit instruction ● Schema-based instruction ● Use visuals ● Use manipulatives ● Use a multi-sensory approach ● Guided notes and/or scaffold outline for written assignments ● Introduce key vocabulary before lesson ● Lesson taught again using a differentiated approach Small group or Individual instruction (Tier III) ● Intensive instruction ● Practice fact retrieval ● Lesson taught again using a differentiated approach ● Allow answers to be given orally or dictated ● Use visuals / Anchor Charts
Students with IEPs and 504 Plans	<u>Universal Design for Learning and Access for Students with Disabilities</u> <u>Movematics (dyscalculia.org)</u> Follow all the IEP/504 plan modifications Allow extra time to complete assignments or tests Guided notes and/or scaffold outline for written assignments Work in a small group Solidify and refine concepts through repetition

	Allow answers to be given orally or dictated Use multi-sensory teaching approaches Utilize assistive technology and materials Use large print books, Braille, or books on CD (digital text) Text to speech/audio recorded selections Amplification system as needed Leveled texts according to ability Fine motor skill stations embedded in rotation as needed Modified or constrained spelling word lists Provide anchor charts with high frequency words and phonemic patterns Students might need guidance as to what actions happen during partner work time. Invite students to discuss what it looks like when they are working with their partner while exploring the connecting cubes. Generate a list of shared expectations for partner work. Invite students to plan a strategy before they begin working. If time allows, invite students to share their plan with a partner. Some students may benefit from feedback that emphasizes effort and time on task. For example, moving counters in the squares of the 5-frame or filling the 5-frame with one counter in each square.
Multi-language Learners	<u>Mathematical Language Development and Access for English Learners</u> Work toward longer passages as skills in English increase

	Use visuals Introduce/highlight key vocabulary before lesson Teacher models reading aloud daily Provide peer tutoring Utilize translation supports including bilingual dictionary, translation device, assistive technology, and peers Guided notes and/or scaffold outline for written assignments Provide students with English Learner leveled readers Support recommended by the grade specific “can do” list outlined by WIDA, including Key Uses Edition and Descriptor Name Charts Create a visual display of what students want to do or make with the connecting cubes. As students share ideas, include a drawing or annotation to illustrate connections. Invite students to use gestures as they count.
Accelerating Students	<u>Accelerating Students Guide</u> Create an enhanced set of introductory activities (e.g. advance organizers, concept maps, concept puzzles) Provide options, alternatives and choices to differentiate and broaden the curriculum Organize and offer flexible small group learning activities Provide whole group enrichment explorations

	Teach cognitive and methodological skills Use center, stations, or contracts Organize integrated problem-solving simulations Propose interest-based extension activities Expose students to beyond level texts.
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(NJDOE, 2024)

Unit 8: Pythagorean Theorem and Irrational Numbers

Unit Goal: In this unit, students work with geometric and symbolic representations of square and cube roots. They understand and use notation such as $\sqrt{2}$ and $\sqrt[3]{5}$ for square and cube roots. They understand the terms “rational number” and “irrational number,” using long division to express fractions as decimals. They use their understanding of fractions to plot rational numbers on the number line and their understanding of approximation of irrationals by rationals to approximate the number-line location of a given irrational. Students learn (without proof) that $\sqrt{2}$ is irrational. They understand two proofs of the Pythagorean Theorem—an algebraic proof that involves manipulation of two expressions for the same area and a geometric proof that involves decomposing and rearranging two squares. They use the Pythagorean Theorem in two and three dimensions, e.g., to determine lengths of diagonals of rectangles and right rectangular prisms, and to estimate distances between points in the coordinate plane.

Duration: 4 weeks

Essential Questions:

- How do geometric and symbolic representations of square and cube roots help us solve problems related to area and volume in geometry?
- What distinguishes rational numbers from irrational numbers, and how can we use long division to express fractions as decimals?
- How does our understanding of fractions enable us to accurately plot rational numbers on the number line, and how do we approximate the location of irrational numbers?
- What are the significance and implications of the Pythagorean Theorem in geometry, and why is it important to understand both geometric and algebraic proofs of the theorem?

- How can we apply the Pythagorean Theorem in practical situations, such as determining the diagonals of rectangles or estimating distances between points in the coordinate plane?
- What are the limitations of rational approximations for irrational numbers, and how do these impact our understanding of geometric concepts and measurements?
- Why is it significant that the square root of 2 is irrational, and what are the implications of this fact in mathematics and beyond?
- How does our understanding of the Pythagorean Theorem extend to three-dimensional space, and what geometric relationships does it help us uncover within right rectangular prisms and other solid shapes?

Enduring Understandings:

- Square and cube roots provide mathematical tools for finding the side lengths of squares and cubes, and their geometric and symbolic representations aid in solving problems involving area and volume.
- Rational numbers, expressed as fractions or decimals, can be plotted on the number line, while irrational numbers require approximation.
- The Pythagorean Theorem offers two distinct yet interconnected proofs—a geometric proof involving square areas and an algebraic proof involving manipulation of expressions—and serves as a foundational principle in geometry and trigonometry.
- The Pythagorean Theorem has practical applications in both two and three dimensions, allowing for the determination of lengths, distances, and relationships within geometric shapes.

Key NJSLS Strands:

- **8.EE.A** Work with radicals and integer exponents.
 - **8.EE.A.2** Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$ where p is a positive rational number.
- **8.G.B** Understand and apply the Pythagorean Theorem.
 - **8.G.B.6** Explain a proof of the Pythagorean Theorem and its converse.
 - **8.G.B.7** Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
 - **8.G.B.8** Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.
- **8.F.B** Use functions to model relationships between quantities.
- **8.NS.A** Know that there are numbers that are not rational and approximate them by rational numbers.

- **8.NS.A.1** Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually and convert a decimal expansion which repeats eventually into a rational number.
- **8.NS.A.2** Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2).

Interdisciplinary Connections

- **NJSLSA.R7** Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words.
- **NJSLSA.SL1** Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively.
- **NJSLSA.SL4** Present information, findings, and supporting evidence such that listeners can follow the line of reasoning and the organization, development, and style are appropriate to task, purpose, and audience.
- **MS-LS2-2** In this statistics unit students construct an explanation that predicts patterns of interactions among organisms.

21st-Century Skills

- **9.1.8.FP.2** Evaluate the role of emotions, attitudes, and behavior (rational and irrational) in making financial decisions.

Career Ready Practices

- **CRP2** Apply appropriate academic and technical skills.
- **CRP8** Utilize critical thinking to make sense of problems and persevere in solving them.
- **CRP11** Use technology to enhance productivity.

Technology Integration

- **9.4.8.TL.6** Collaborate to develop and publish work that provides perspectives on a real-world problem.

Skills	Instructional Topics	Assessment	Resources	Vocabulary
Identify numbers that are not rational are called irrational	The Areas of Squares and Their Side Lengths	Formative Assessments:	Materials:	square root
Explain that every number has a decimal expansion	Side Lengths and Areas	Diagnostic Assessment	blank paper	square (of a number)
Show that rational numbers have decimal expansions that either terminate in zeros or repeats eventually	Rational and Irrational Numbers	Mid-topic Checkpoint	colored pencils	irrational number
Convert a repeating decimal to a rational number	Square Roots on the Number Line	Lesson Quizzes	compasses	square root symbol
Estimate the value of irrational numbers using rational approximations	Reasoning About Square Roots	Lesson Self-assessment	copies of blackline master	rational number
Use rational approximations of irrational numbers to compare their size	The Pythagorean Theorem	Teacher Observations	dried linguine pasta	diagonal
Use rational approximations of irrational numbers to locate them on a number	Finding Side Lengths of Triangles	Class Discussions	Four-function calculators	decimal approximation
	A Proof of the Pythagorean Theorem	Classwork/ Homework	tracing paper	square root
	Finding Unknown Side Lengths	Exit Slips/Google Form	graph paper	square root symbol
	The Converse	Summative Assessments:	colored pencils	Pythagorean Theorem
	Applications of the Pythagorean Theorem	Topic/Unit Assessments	scissors	hypotenuse
	Finding Distances in the	Constructed Response Questions	index cards	legs
		Projects	graduated cylinders	right triangle
		Cumulative Tests	graph paper	converse of the Pythagorean Theorem
		Performance Assessments	isometric graph paper	Pythagorean Theorem
				edge length

line Define a function is a rule that assigns to each input exactly one output the graph of a function is the set of ordered pairs consisting of an input and the corresponding output Use square root and cube root symbols to represent solutions to equations in the form $x^2 = p$ and $x^3 = p$ Evaluate square roots of small perfect squares and cube roots of small perfect cubes Identify that $\sqrt{2}$ is an irrational number Explain how the Pythagorean Theorem states that the square of the hypotenuse of a right triangle is equal to the sum of the squares	Coordinate Plane Side Lengths and Volumes of Cubes Edge Lengths and Volumes Cube Roots Decimal Representation of Rational and Irrational Numbers Decimal Representations of Rational Numbers Infinite Decimal Expansions When Is the Same Size Not the Same Size?	Alternate and Benchmark Assessments: Progress Monitoring Assessments Cumulative/Benchmark Assessments	long straightedge measuring tapes protractors rulers rulers marked with centimeters rulers marked with inches scientific calculators scissors spherical objects stopwatches straightedges string tape tools for creating a visual display	hypotenuse legs cube root cube root edge length repeating decimal decimal representation finite decimal expansion infinite decimal expansion irrational number repeating decimal
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of the other two sides Explain a proof of the Pythagorean Theorem Explain a proof of the converse of the Pythagorean Theorem Apply the Pythagorean Theorem to find the distance between two points in a coordinate system Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in two-dimensional figures Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in three-dimensional figures Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world problems			toothpicks, pencils, straws, or other objects tracing paper Activities: Root Realm Discovery Rationality Rumble Decimal Conversion Crusade Line of Numbers The Pythagorean Proof Parade The Pythagorean Puzzle Irrational Quest Radical Investigation Mentor Texts: <i>What's Your Angle, Pythagoras?</i> by Julie Ellis <i>Pythagoras and the Ratios: A Math Adventure</i> by Julie Ellis	
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Modifications and Accommodations	
Tier II and III Strategies	Small group instruction (Tier II) ● Explicit instruction ● Schema-based instruction ● Use visuals ● Use manipulatives ● Use a multi-sensory approach ● Guided notes and/or scaffold outline for written assignments ● Introduce key vocabulary before lesson ● Lesson taught again using a differentiated approach Small group or Individual instruction (Tier III) ● Intensive instruction ● Practice fact retrieval ● Lesson taught again using a differentiated approach ● Allow answers to be given orally or dictated ● Use visuals / Anchor Charts
Students with IEPs and 504 Plans	<u>Universal Design for Learning and Access for Students with Disabilities</u> <u>Movematics (dyscalculia.org)</u> Follow all the IEP/504 plan modifications Allow extra time to complete assignments or tests Guided notes and/or scaffold outline for written assignments Work in a small group Solidify and refine concepts through repetition

	Allow answers to be given orally or dictated Use multi-sensory teaching approaches Utilize assistive technology and materials Use large print books, Braille, or books on CD (digital text) Text to speech/audio recorded selections Amplification system as needed Leveled texts according to ability Fine motor skill stations embedded in rotation as needed Modified or constrained spelling word lists Provide anchor charts with high frequency words and phonemic patterns Students might need guidance as to what actions happen during partner work time. Invite students to discuss what it looks like when they are working with their partner while exploring the connecting cubes. Generate a list of shared expectations for partner work. Invite students to plan a strategy before they begin working. If time allows, invite students to share their plan with a partner. Some students may benefit from feedback that emphasizes effort and time on task. For example, moving counters in the squares of the 5-frame or filling the 5-frame with one counter in each square.
Multi-language Learners	<u>Mathematical Language Development and Access for English Learners</u> Work toward longer passages as skills in English increase

	Use visuals Introduce/highlight key vocabulary before lesson Teacher models reading aloud daily Provide peer tutoring Utilize translation supports including bilingual dictionary, translation device, assistive technology, and peers Guided notes and/or scaffold outline for written assignments Provide students with English Learner leveled readers Support recommended by the grade specific “can do” list outlined by WIDA, including Key Uses Edition and Descriptor Name Charts Create a visual display of what students want to do or make with the connecting cubes. As students share ideas, include a drawing or annotation to illustrate connections. Invite students to use gestures as they count.
Accelerating Students	<u>Accelerating Students Guide</u> Create an enhanced set of introductory activities (e.g. advance organizers, concept maps, concept puzzles) Provide options, alternatives and choices to differentiate and broaden the curriculum Organize and offer flexible small group learning activities Provide whole group enrichment explorations

	Teach cognitive and methodological skills Use center, stations, or contracts Organize integrated problem-solving simulations Propose interest-based extension activities Expose students to beyond level texts.
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(NJDOE, 2024)

Unit 9: Integrating Math Skills

Unit Goal: In this unit, students solve complex problems. First, they consider tessellations of the plane, understanding and using the terms “tessellation” and “regular tessellation” in their work, and using properties of shapes (for example, the sum of the interior angles of a quadrilateral is 360 degrees) to make inferences about regular tessellations. Next, they investigate relationships of temperature and latitude, climate, season, cloud cover, or time of day. In particular, they use scatter plots and lines of best fit to investigate the question of modeling temperature as a function of latitude.

Duration: 2 weeks

Essential Questions:

- How do tessellations of the plane reflect the properties of the shapes used, and what distinguishes a regular tessellation from other tessellations?
- What geometric properties allow certain shapes to tessellate the plane, and how can we use these properties to predict and classify tessellations?
- How do factors such as temperature, latitude, climate, season, cloud cover, and time of day interact and influence each other?
- How can scatter plots help us visualize relationships between variables, and what do lines of best fit reveal about these relationships?
- What mathematical techniques can we use to model temperature as a function of latitude, and how do these models help us understand global climate patterns?

- How do real-world factors such as latitude and climate affect our everyday lives, and how can we use mathematical models to better understand and predict these effects?
- How do changes in factors like cloud cover or season impact temperature patterns, and how can we use data analysis to explore these relationships?
- What are the limitations of modeling complex phenomena like climate and weather, and how can we account for uncertainty in our analyses and predictions?

Enduring Understandings:

- Tessellations of the plane involve repeating patterns of shapes to cover a surface without any gaps or overlaps, and understanding regular tessellations requires knowledge of the properties of shapes.
- Properties of geometric shapes, such as the sum of interior angles of polygons, can be used to analyze and classify tessellations.
- Various factors, including temperature, latitude, climate, season, cloud cover, and time of day, influence each other in complex ways.
- Scatter plots and lines of best fit are tools used to model and analyze relationships between variables, such as temperature and latitude.

Key NJSLS Strands:

- **8.G.A** Understand congruence and similarity using physical models, transparencies, or geometry software.
 - **8.G.A.5** Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.
- **8.F.A** Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.
 - **8.F.A.1** Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.
- **8.F.B** Use functions to model relationships between quantities.
- **8.SP.A** Investigate patterns of association in bivariate data.

Interdisciplinary Connections
● NJSLSA.R7 Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words. ● NJSLSA.SL1 Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively ● NJSLSA.SL4 Present information, findings, and supporting evidence such that listeners can follow the line of reasoning and the organization, development, and style are appropriate to task, purpose, and audience. ● MS-ETS1-1: Students estimate sums and differences while defining the ● criteria and constraints of a design problem with sufficient precision to ensure a ● successful solution, taking into account relevant scientific principles.
21st-Century Skills
● 9.1.8.CR.2 Compare various ways to give back through strengths, passions, goals, and other personal factors.
Career Ready Practices
● CRP2 Apply appropriate academic and technical skills. CRP4: Communicate clearly and effectively with reason. CRP6: Demonstrate creativity and innovation.
Technology Integration
● 9.4.5.CT.3 Describe how digital tools and technology may be used to solve problems.

Skills	Instructional Topics	Assessment	Resources	Vocabulary
Identify the sum of the interior angles of a triangle is 180 degrees	Tessellations of the Plane Regular Tessellations	Formative Assessments: Pre-Topic Assessment	Materials: blank paper colored pencils	tessellation pattern

Explain the measure of an exterior angle of a triangle is equal to the sum of the Identify two remote interior angles Explain when parallel lines are cut by a transversal, corresponding, alternate Explain interior, and alternate exterior angles are congruent Explain if two sets of corresponding angles in two triangles are congruent, then the triangles are similar Use facts about angles to construct an informal argument Define a function is a rule that assigns to each input exactly one output Define the graph of a function is the set of ordered pairs consisting	Tessellating Polygons What Influences Temperature? Plotting the Weather Using and Interpreting a Mathematical Model	Mid-topic Checkpoint Lesson Quizzes Lesson Self-assessment Teacher Observations Class Discussions Classwork/ Homework Exit Slips/Google Form Summative Assessments: Topic/Unit Assessments Constructed Response Questions Projects Cumulative Tests Performance Assessments Alternate and Benchmark Assessments: Progress Monitoring Assessments	compasses copies of blackline master dried linguine pasta Four-function calculators tracing paper graph paper colored pencils scissors index cards graduated cylinders graph paper isometric graph paper long straightedge measuring tapes protractors rulers	regular tessellation regular polygon mathematical model
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of an input and the corresponding output Construct scatter plots Interpret scatter plots to investigate patterns of association between two quantities Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association Construct a function to model a linear relationship between two quantities Determine the rate of change and initial value of a function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph Interpret the rate of change and initial value of a function in terms of		Cumulative/Benchmark Assessments	rulers marked with centimeters rulers marked with inches scientific calculators scissors spherical objects stopwatches straightedges string tape tools for creating a visual display toothpicks, pencils, straws, or other objects tracing paper Activities: Tessellation Exploration Tessellation Analysis Climate Factors Investigation	
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the situation it models			Scatter Plot Analysis Modeling Temperature and Latitude Real-World Climate Patterns Effects of Changing Variables Uncertainty and Limitations Discussion Mentor Texts: <i>Tessellations: The History and Making of Symmetrical Designs</i> by Eric G. Barbour	
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Modifications and Accommodations	
Tier II and III Strategies	Small group instruction (Tier II) ● Explicit instruction ● Schema-based instruction ● Use visuals ● Use manipulatives ● Use a multi-sensory approach ● Guided notes and/or scaffold outline for written assignments ● Introduce key vocabulary before lesson ● Lesson taught again using a differentiated approach Small group or Individual instruction (Tier III) ● Intensive instruction ● Practice fact retrieval ● Lesson taught again using a differentiated approach

	• Allow answers to be given orally or dictated • Use visuals / Anchor Charts
Students with IEPs and 504 Plans	<u>Universal Design for Learning and Access for Students with Disabilities</u> <u>Movematics (dyscalculia.org)</u> Follow all the IEP/504 plan modifications Allow extra time to complete assignments or tests Guided notes and/or scaffold outline for written assignments Work in a small group Solidify and refine concepts through repetition Allow answers to be given orally or dictated Use multi-sensory teaching approaches Utilize assistive technology and materials Use large print books, Braille, or books on CD (digital text) Text to speech/audio recorded selections Amplification system as needed Leveled texts according to ability Fine motor skill stations embedded in rotation as needed Modified or constrained spelling word lists Provide anchor charts with high frequency words and phonemic patterns

	Students might need guidance as to what actions happen during partner work time. Invite students to discuss what it looks like when they are working with their partner while exploring the connecting cubes. Generate a list of shared expectations for partner work. Invite students to plan a strategy before they begin working. If time allows, invite students to share their plan with a partner. Some students may benefit from feedback that emphasizes effort and time on task. For example, moving counters in the squares of the 5-frame or filling the 5-frame with one counter in each square.
Multi-language Learners	<u>Mathematical Language Development and Access for English Learners</u> Work toward longer passages as skills in English increase Use visuals Introduce/highlight key vocabulary before lesson Teacher models reading aloud daily Provide peer tutoring Utilize translation supports including bilingual dictionary, translation device, assistive technology, and peers Guided notes and/or scaffold outline for written assignments Provide students with English Learner leveled readers Support recommended by the grade specific “can do” list outlined by WIDA, including Key Uses Edition and Descriptor Name Charts Create a visual display of what students want to do or make with the connecting cubes.

	As students share ideas, include a drawing or annotation to illustrate connections. Invite students to use gestures as they count.
Accelerating Students	<u>Accelerating Students Guide</u> Create an enhanced set of introductory activities (e.g. advance organizers, concept maps, concept puzzles) Provide options, alternatives and choices to differentiate and broaden the curriculum Organize and offer flexible small group learning activities Provide whole group enrichment explorations Teach cognitive and methodological skills Use center, stations, or contracts Organize integrated problem-solving simulations Propose interest-based extension activities Expose students to beyond level texts.

(NJDOE, 2024)

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References

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