

Englewood on the Palisades Charter School

Mathematics Curriculum

Grade 7

Aligned With the 2023 New Jersey Student Learning Standards

The Vision for Mathematics Education in New Jersey

A New Jersey education in Mathematics builds quantitatively and analytically literate citizens prepared to meet the demands of college and career, and to engage productively in an information-driven society. All students will have access to a high-quality mathematics education that fosters a population that:

- leverages data in decision-making and as a lens for discussing, analyzing, and responding to practical questions.
- persists to make sense of and model problems arising in everyday life, society, and the workplace.
- thinks critically and strategically to assess quantitative relationships and to solutions to complex problems.
- employs precise reasoning and constructs viable arguments to deduce conclusions, recognize false statements and assess peers' reasoning.
- interprets, evaluates and critiques the mathematics embedded in social, scientific and commercial systems, as well as the claims made in the private and public sectors.
- communicates precisely when conveying, representing, and justifying both qualitative and quantitative perspectives.

(NJ DOE, 2024)

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

Standard	Mathematically Proficient Students ...
1. Make sense of problems and persevere in solving them	start by explaining to themselves the meaning of a problem and looking for entry points to its solution. check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?”
2. Reason abstractly and quantitatively	make sense of quantities and their relationships in problem situations.
3. Construct viable arguments and critique the reasoning of others	understand and use stated assumptions, definitions, and previously established results in constructing arguments.
4. Model with mathematics	apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later.
5. Use appropriate tools strategically	are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations.
6. Attend to precision	try to communicate precisely to others.
7. Look for and make use of structure	look closely to discern a pattern or structure

8. Look for and express regularity in repeated reasoning	notice if calculations are repeated, and look both for general methods and for shortcuts.
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(NJ DOE, 2024)

New Jersey Legislative Statutes and Administrative Code Mathematics

Amistad Law: *N.J.S.A. 18A 52:16A-88*

[Radical Math](#): includes lessons on topics for social justice in mathematics
[African American Mathematician Bios](#)

Holocaust Law: *N.J.S.A. 18A:35-28*

<https://www.bbc.com/future/article/20191031-hilda-geiringer-mathematician-who-fled-the-nazis>

LGBT and Disabilities Law: *N.J.S.A. 18A:35-4.35*

<https://prideinstem.org/>
[LGBTQA+ Mathematician Bios](#)
<https://www.ngpf.org/blog/math/math-monday-celebrating-disabled-mathematicians/>
[Mathematicians with Disabilities](#)

Diversity & Inclusion: *N.J.S.A. 18A:35-4.36a*

<http://www.ams.org/about-us/diversity>
<https://mathematicallygiftedandblack.com/>

Standards in Action: *Climate Change*

New Jersey became the first state in the nation to include climate change across content areas with the adoption of the 2020 New Jersey Student Learning Standards (NJSLS). The goal of inclusion of climate change education implementation is to foster generations of New Jersey students that can analyze, question, interpret, to think independently, and bring critical deduction to fulfill, and to lead in jobs created by burgeoning industries of the future green economy.

Suggestions for how to incorporate climate change examples into math instruction will be noted in **green text** throughout the units.

Math Curriculum

Grade Seven

This seventh grade math curriculum aligns with the 2023 New Jersey Student Learning Standards (NJSLS) and provides a solid foundation for learners to develop understanding in four critical areas this year. Instruction in grade three focuses broadly on four critical areas:

- developing understanding of and applying proportional relationships;
- developing understanding of operations with rational numbers and working with expressions and linear equations;
- solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and
- drawing inferences about populations based on samples.

Domains and Cluster Areas

- **Ratio and Proportional Relationships**
 - Analyze proportional relationships and use them to solve real-world and mathematical problems.
- **The Number System**

- Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.
- **Expressions and Equations**
 - Use properties of operations to generate equivalent expressions.
 - Solve real-life and mathematical problems using numerical and algebraic expressions and equations.
- **Geometry**
 - Draw, construct and describe geometrical figures and describe the relationships between them.
 - Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.
- **Statistics and Probability**
 - Use random sampling to draw inferences about a population.
 - Draw informal comparative inferences about two populations.
 - Investigate chance processes and develop, use, and evaluate probability models.

Emphases are given at the cluster level. Refer to the New Jersey Student Learning Standards for Mathematics for the specific standards that fall within each cluster. Even though the clusters have different levels of emphasis, this does not mean that standards can be omitted. All standards must be included in instruction throughout a grade level to ensure that students will be prepared for instruction at higher grade levels.

Indicator	Type	Cluster Heading
7.RP.A	M	Analyze proportional relationships and use them to solve real-world and mathematical problems

7.NS.A	M	Apply and extend previous understandings of operations with fractions to add, subtract, multiply and divide rational numbers
7.EE.A	M	Use properties of operations to generate equivalent expressions
7.EE.B	M	Solve real-life and mathematical problems using numerical and algebraic expressions and equations
7.G.A	A	Draw, construct and describe geometrical figures and describe the relationships between them
7.G.B	A	Solve real-life and mathematical problems involving angle measure, area, surface area and volume
7.SP.A	S	Use random sampling to draw inferences about a population
7.SP.B	A	Draw informal comparative inferences about two populations
7.SP.C	S	Investigate chance processes and develop, use, and evaluate probability models.

Key: M — Major, S — Supporting Clusters, A — Additional Clusters

Required Fluencies in Grade 7:

- 7.EE.B.4a Solve equations of the form $px+q=r$ and $p(x+q)=r$.

(NJDOE, 2024)

Grade 7 Units and Pacing	
Unit 1: Scale Drawings	3 weeks
Unit 2: Introducing Proportional Relationships	4 weeks
Unit 3: Measuring Circles	3 weeks
Unit 4: Proportional Relationships and Percentages	4 weeks
Unit 5: Rational Number Arithmetic	4 weeks
Unit 6: Expressions, Equations, and Inequalities	5 weeks
Unit 7: Angles, Triangles, and Prisms	4 weeks
Unit 8: Probability and Sampling	5 weeks
Unit 9: Integrating Math Skills	3 weeks
Total Number of Weeks:	35 weeks

Unit 1: Scale Drawings

Unit Goal: In this unit, students learn to understand and use the terms “scaled copy,” “to scale,” “scale factor,” “scale drawing,” and “scale,” and recognize when two pictures or plane figures are or are not scaled copies of each other. They use tables to reason about measurements in scaled copies, and recognize that angle measures are preserved in scaled copies, but lengths are scaled by a scale factor and areas by the square of the scale factor. They make, interpret, and reason about scale drawings. These include maps and floor plans that have scales with and without units.

Duration: 3 weeks

Essential Questions:

- How can understanding terms like "scaled copy," "scale factor," and "scale drawing" enhance our ability to interpret and create representations of real-world objects and designs?
- How does the concept of scale factor influence the relationship between measurements, angles, lengths, and areas in scaled copies?
- What role do tables play in reasoning about measurements and relationships in scaled drawings, and how can they aid in problem-solving?
- How does recognizing and interpreting scales in maps and floor plans contribute to our understanding of spatial relationships and design concepts?

Enduring Understandings:

- Understanding terms like "scaled copy," "scale factor," and "scale drawing" enables us to accurately represent and interpret objects and designs at different proportions, facilitating communication and problem-solving in various contexts.
- In scaled copies, angle measures remain constant, while lengths are scaled proportionally by a scale factor and areas are scaled by the square of the scale factor, illustrating the relationship between geometric properties and scale transformations.
- Tables provide a structured framework for organizing and reasoning about measurements in scaled drawings, allowing us to analyze relationships and make informed decisions in design and problem-solving tasks.
- Recognizing and interpreting scales in maps and floor plans allows us to understand spatial relationships, navigate physical environments, and make informed decisions about layout and design, enhancing our spatial reasoning skills and visual literacy.

Key NJSLS Strands:

- **7.RP.A** Analyze proportional relationships and use them to solve real-world and mathematical problems.
 - **7.RP.A.2** Recognize and represent proportional relationships between quantities.
 - **7.RP.A.2.a** Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
 - **7.RP.A.2.b** Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

- **7.RP.A.3** Count forward beginning from a given number within the known sequence (instead of having to begin at 1).
- **7.G.A** Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).
 - **7.G.A.1** Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
- **7.G.B** Solve real-life and mathematical problems involving angle measure, area, surface area, and volume
 - **7.G.B.4** Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.
 - **7.G.B.6** Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. **Climate Change Example: Students may solve real-world problems involving area, surface area, and volume related to deforestation and increasing livestock farming as key contributors to climate change.**

Interdisciplinary Connections

- **NJSLSA.R7** Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words.
- **NJSLSA.SL1** Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively.
- **NJSLSA.SL4** Present information, findings, and supporting evidence such that listeners can follow the line of reasoning and the organization, development, and style are appropriate to task, purpose, and audience.
- **MS-ETS1-1** Students estimate sums and differences while defining the criteria and constraints of a design problem with sufficient precision to ensure a successful solution, taking into account relevant scientific principles

21st-Century Skills

- **9.2.4.A.4** Explain why knowledge and skills acquired in the elementary grades lay the foundation for future academic and career success.

Career Ready Practices
● CRP2 Apply appropriate academic and technical skills. ● CRP8 Utilize critical thinking to make sense of problems and persevere in solving them. ● CRP11 Use technology to enhance productivity.
Technology Integration
● 9.4.8.TL.3 Select appropriate tools to organize and present information digitally. ● 9.4.8.TL.6 Collaborate to develop and publish work that provides perspectives on a real-world problem.

Skills	Instructional Topics	Assessment	Resources	Vocabulary
Compute unit rates involving ratios of fractions (complex fractions) in quantities measured in like or different units. Decide whether two quantities show a proportional relationship by testing for equivalent ratios in a table. Decide whether two quantities show a proportional relationship by graphing on a coordinate plane and observing whether the	Scaled Copies Corresponding Parts and Scale Factors Making Scaled Copies Scaled Relationships The Size of the Scale Factor Scaling and Area Scale Drawings Scale Drawings and Maps Creating Scale	Formative Assessments: Diagnostic Assessment Mid-topic Checkpoint Lesson Quizzes Lesson Self-assessment Teacher Observations Class Discussions Classwork/ Homework Exit Slips/Google Form Summative Assessments:	Materials: blank paper coins colored pencils compasses cylindrical household items empty toilet paper roll four-function calculators fruits or vegetables tracing paper	scaled copy original polygon corresponding scale factor figure segment quadrilateral measurement distance corresponding

graph is a straight line through the origin. Identify the constant of proportionality (unit rate) in equations and verbal descriptions of proportional relationships. Identify the constant of proportionality (unit rate) in tables, graphs, and diagrams. Represent proportional relationships by equations using the constant of proportionality (unit rate). Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate. Solve problems involving scale drawings of geometric	Drawings Changing Scales in Scale Drawings Scales without Units Units in Scale Drawings Draw It to Scale	Topic/Unit Assessments Constructed Response Questions Projects Cumulative Tests Performance Assessments Alternate and Benchmark Assessments: Progress Monitoring Assessments Cumulative/Benchmark Assessments	graph paper colored pencils scissors index cards rulers protractor glue or glue sticks graph paper grocery store circulars index cards internet-enabled device knife maps or satellite images of the school grounds markers materials assembled from the blackline master measuring cup	reciprocal area one-dimensional two-dimensional squared scale drawing scale represent actual three-dimensional scaled copy estimate travel constant speed floorplan appropriate dimension scale without units ___ to ___
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figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. Draw informal comparative inferences about two populations by using the measures of center (mean and median) and measures of variability (interquartile range and mean absolute deviation) from random samples. Solve real-world and mathematical problems involving volume and surface area of three-dimensional objects composed of cubes and right prisms.			measuring spoons measuring tapes measuring tools metal paper fasteners (brass brads) meter sticks metric and customary unit conversion charts mixing containers number cubes pain paper bags paper clips paper cups paper plates pattern blocks pre-assembled polyhedra protractors receipt tape	equivalent scales
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			recipes rulers rulers marked with centimeters rulers marked with inches scissors small disposable cups snap cubes sticky notes stopwatches straightedges straws string tape tools for creating a visual display trundle wheels water yardsticks	
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			Activities: Scale Factor Exploration Identifying Scaled Copies Corresponding Parts Matching Scale Factor Investigation Creating Scaled Models Digital Scaled Drawings Map Scale Analysis Scale Drawing Challenge Scaling Up and Down Modeling Scale Changes Unit-less Scale Exploration Scaling Challenges Unit Conversion in Scale Drawings	
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			Real-world Applications Draw to Scale Challenge Scale Drawing Gallery Mentor Texts: <i>The Westing Game</i> by Ellen Raskin <i>Chasing Vermeer</i> by Blue Balliet	
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Modifications and Accommodations	
Tier II and III Strategies	Small group instruction (Tier II) ● Explicit instruction ● Schema-based instruction ● Use visuals ● Use manipulatives ● Use a multi-sensory approach ● Guided notes and/or scaffold outline for written assignments ● Introduce key vocabulary before lesson ● Lesson taught again using a differentiated approach Small group or Individual instruction (Tier III) ● Intensive instruction ● Practice fact retrieval ● Lesson taught again using a differentiated approach ● Allow answers to be given orally or dictated ● Use visuals / Anchor Charts

Students with IEPs and 504 Plans	<u>Universal Design for Learning and Access for Students with Disabilities</u> <u>Movematics (dyscalculia.org)</u> Follow all the IEP/504 plan modifications Allow extra time to complete assignments or tests Guided notes and/or scaffold outline for written assignments Work in a small group Solidify and refine concepts through repetition Allow answers to be given orally or dictated Use multi-sensory teaching approaches Utilize assistive technology and materials Use large print books, Braille, or books on CD (digital text) Text to speech/audio recorded selections Amplification system as needed Leveled texts according to ability Fine motor skill stations embedded in rotation as needed Modified or constrained spelling word lists Provide anchor charts with high frequency words and phonemic patterns Students might need guidance as to what actions happen during partner work time. Invite students to discuss what it looks like when they are working with their partner while exploring the connecting cubes. Generate a list of shared expectations for partner work.
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	Invite students to plan a strategy before they begin working. If time allows, invite students to share their plan with a partner. Some students may benefit from feedback that emphasizes effort and time on task. For example, moving counters in the squares of the 5-frame or filling the 5-frame with one counter in each square.
Multi-language Learners	<u>Mathematical Language Development and Access for English Learners</u> Work toward longer passages as skills in English increase Use visuals Introduce/highlight key vocabulary before lesson Teacher models reading aloud daily Provide peer tutoring Utilize translation supports including bilingual dictionary, translation device, assistive technology, and peers Guided notes and/or scaffold outline for written assignments Provide students with English Learner leveled readers Support recommended by the grade specific “can do” list outlined by WIDA, including Key Uses Edition and Descriptor Name Charts Create a visual display of what students want to do or make with the connecting cubes. As students share ideas, include a drawing or annotation to illustrate connections. Invite students to use gestures as they count.

Accelerating Students	<u>Accelerating Students Guide</u> Create an enhanced set of introductory activities (e.g. advance organizers, concept maps, concept puzzles) Provide options, alternatives and choices to differentiate and broaden the curriculum Organize and offer flexible small group learning activities Provide whole group enrichment explorations Teach cognitive and methodological skills Use center, stations, or contracts Organize integrated problem-solving simulations Propose interest-based extension activities Expose students to beyond level texts.
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(NJDOE, 2024)

Unit 2: Introducing Proportional Rates

Unit Goal: In this unit, students learn to understand and use the terms “proportional,” “constant of proportionality,” and “proportional relationship,” and recognize when a relationship is or is not proportional. They represent proportional relationships with tables, equations, and graphs. Students use these terms and representations in reasoning about situations that involve constant speed, unit pricing, and measurement conversions.
Duration: 4 weeks
Essential Questions: How do we recognize and distinguish between proportional and non-proportional relationships in real-world situations?

- What role do tables, equations, and graphs play in representing and analyzing proportional relationships?
- How does understanding the constant of proportionality contribute to our ability to reason about situations involving constant speed, unit pricing, and measurement conversions?

Enduring Understandings:

- Understanding terms like "proportional," "constant of proportionality," and "proportional relationship" enables us to identify and interpret relationships where changes in one quantity correspond to changes in another quantity at a consistent rate.
- Representing proportional relationships with tables, equations, and graphs provides multiple ways to visualize and analyze the relationship between quantities, enhancing our ability to make predictions and solve problems.
- Recognizing and applying the constant of proportionality allows us to reason about real-world scenarios involving constant speed, unit pricing, and measurement conversions, providing a foundation for making informed decisions and solving practical problems.

Key NJSLS Strands:

- **7.G.A** Draw, construct, and describe geometrical figures and describe the relationships between them.
 - **7.G.A.1** Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
- **7.G.B** Solve real-life and mathematical problems involving angle measure, area, surface area, and volume
 - **7.G.B.6** Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
- **7.RP.A** Analyze proportional relationships and use them to solve real-world and mathematical problems.
 - **7.RP.A.1** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.
 - **7.RP.A.2** Recognize and represent proportional relationships between quantities.
 - **7.RP.A.2.a** Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
 - **7.RP.A.2.b** Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
 - **7.RP.A.2.c** Represent proportional relationships by equations.

- **7.RP.A.2.d** Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate.
- **7.EE.A** Use properties of operations to generate equivalent expressions.

Interdisciplinary Connections

- **NJSLSA.R7** Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words.
- **NJSLSA.SL1** Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively.
- **NJSLSA.SL4** Present information, findings, and supporting evidence such that listeners can follow the line of reasoning and the organization, development, and style are appropriate to task, purpose, and audience.
- **MS-PS3-3** Apply scientific principles to design, construct, and test a device that either minimizes or maximizes thermal energy transfer.

21st-Century Skills

- **9.2.8.CAP.3** Explain how career choices, educational choices, skills, economic conditions, and personal behavior affect income.

Career Ready Practices

- **CRP2** Apply appropriate academic and technical skills.
- **CRP8** Utilize critical thinking to make sense of problems and persevere in solving them.
- **CRP11** Use technology to enhance productivity.

Technology Integration

- **9.4.8.TL.3** Select appropriate tools to organize and present information digitally.
- **9.4.8.TL.6** Collaborate to develop and publish work that provides perspectives on a real-world problem.

Skills	Instructional Topics	Assessment	Resources	Vocabulary
Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. Solve real-world and mathematical problems involving volume and surface area of three-dimensional objects composed of cubes and right prisms. Compute unit rates involving ratios of fractions (complex fractions) in quantities measured in like or different units. Decide whether two quantities show a proportional relationship by testing for equivalent	Representing Proportional Relationships with Tables More about Constant of Proportionality Representing Proportional Relationships with Equations Proportional Relationships and Equations Two Equations for Each Relationship Using Equations to Solve Problems Comparing Proportional and Nonproportional Relationships Comparing Relationships with Tables Comparing	Formative Assessments: Diagnostic Assessment Mid-topic Checkpoint Lesson Quizzes Lesson Self-assessment Teacher Observations Class Discussions Classwork/ Homework Exit Slips/Google Form Summative Assessments: Topic/Unit Assessments Constructed Response Questions Projects Cumulative Tests Performance Assessments	Materials: blank paper coins colored pencils compasses cylindrical household items empty toilet paper roll four-function calculators fruits or vegetables tracing paper graph paper colored pencils scissors index cards rulers	constant of proportionality proportional relationship value equivalent ratios row column ___ is proportional to ___ relate constant reciprocal per equation quotient ___ is proportional to ___

ratios in a table. Decide whether two quantities show a proportional relationship by graphing on a coordinate plane and observing whether the graph is a straight line through the origin. Identify the constant of proportionality (unit rate) in equations and verbal descriptions of proportional relationships. Identify the constant of proportionality (unit rate) in tables, graphs, and diagrams. Apply the properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. Rewriting an expression in different forms can clarify the problem and how the quantities are	Relationships with Equations Solving Problems about Proportional Relationships Representing Proportional Relationships with Graphs Introducing Graphs of Proportional Relationships Interpreting Graphs of Proportional Relationships Using Graphs to Compare Relationships Two Graphs for Each Relationship Four Representations	Alternate and Benchmark Assessments: Progress Monitoring Assessments Cumulative/Benchmark Assessments	protractors glue or glue sticks graph paper grocery store circulars index cards internet-enabled device knife maps or satellite images of the school grounds markers materials assembled from the blackline master measuring cup measuring spoons measuring tapes measuring tools metal paper fasteners (brass brads)	steady situation constant of proportionality proportional relationship constant origin coordinate plane plot quantity axes coordinates x-coordinate y-coordinate origin axes reasonable
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related.			meter sticks metric and customary unit conversion charts mixing containers number cubes pain paper bags paper clips paper cups paper plates pattern blocks pre-assembled polyhedra protractors receipt tape recipes rulers rulers marked with centimeters	
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			rulers marked with inches scissors small disposable cups snap cubes sticky notes stopwatches straightedges straws string tape tools for creating a visual display trundle wheels water Yardsticks Activities:	
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			Table Exploration Table Matching Constant of Proportionality Discovery Real-world Constant of Proportionality Equation Creation Equation Comparison Two Equations Challenge Equation Problem-solving Stations Comparing Relationships Task Cards Real-world Scenarios Analysis Graph Exploration Graph Matching Mentor Texts: <i>Counting by 7s</i> by Holly Goldberg Sloan	
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Modifications and Accommodations	
Tier II and III Strategies	Small group instruction (Tier II) ● Explicit instruction ● Schema-based instruction ● Use visuals ● Use manipulatives ● Use a multi-sensory approach ● Guided notes and/or scaffold outline for written assignments ● Introduce key vocabulary before lesson ● Lesson taught again using a differentiated approach Small group or Individual instruction (Tier III) ● Intensive instruction ● Practice fact retrieval ● Lesson taught again using a differentiated approach ● Allow answers to be given orally or dictated ● Use visuals / Anchor Charts
Students with IEPs and 504 Plans	<u>Universal Design for Learning and Access for Students with Disabilities</u> <u>Movematics (dyscalculia.org)</u> Follow all the IEP/504 plan modifications Allow extra time to complete assignments or tests Guided notes and/or scaffold outline for written assignments Work in a small group

	Solidify and refine concepts through repetition Allow answers to be given orally or dictated Use multi-sensory teaching approaches Utilize assistive technology and materials Use large print books, Braille, or books on CD (digital text) Text to speech/audio recorded selections Amplification system as needed Leveled texts according to ability Fine motor skill stations embedded in rotation as needed Modified or constrained spelling word lists Provide anchor charts with high frequency words and phonemic patterns Students might need guidance as to what actions happen during partner work time. Invite students to discuss what it looks like when they are working with their partner while exploring the connecting cubes. Generate a list of shared expectations for partner work. Invite students to plan a strategy before they begin working. If time allows, invite students to share their plan with a partner. Some students may benefit from feedback that emphasizes effort and time on task. For example, moving counters in the squares of the 5-frame or filling the 5-frame with one counter in each square.
Multi-language Learners	<u>Mathematical Language Development and Access for English Learners</u>

	Work toward longer passages as skills in English increase Use visuals Introduce/highlight key vocabulary before lesson Teacher models reading aloud daily Provide peer tutoring Utilize translation supports including bilingual dictionary, translation device, assistive technology, and peers Guided notes and/or scaffold outline for written assignments Provide students with English Learner leveled readers Support recommended by the grade specific “can do” list outlined by WIDA, including Key Uses Edition and Descriptor Name Charts Create a visual display of what students want to do or make with the connecting cubes. As students share ideas, include a drawing or annotation to illustrate connections. Invite students to use gestures as they count.
Accelerating Students	<u>Accelerating Students Guide</u> Create an enhanced set of introductory activities (e.g. advance organizers, concept maps, concept puzzles) Provide options, alternatives and choices to differentiate and broaden the curriculum Organize and offer flexible small group learning activities

	Provide whole group enrichment explorations Teach cognitive and methodological skills Use center, stations, or contracts Organize integrated problem-solving simulations Propose interest-based extension activities Expose students to beyond level texts.
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(NJDOE, 2024)

Unit 3: Measuring Circles

Unit Goal: In this unit, students learn to understand and use the term “circle” to mean the set of points that are equally distant from a point called the “center.” They gain an understanding of why the circumference of a circle is proportional to its diameter, with constant of proportionality π. They see informal derivations of the fact that the area of a circle is equal to π times the square of its radius. Students use the relationships of circumference, radius, diameter, and area of a circle to find lengths and areas, expressing these in terms of π or using appropriate approximations of π to express them numerically.
Duration: 3 weeks
Essential Questions: • What defines a circle, and how is it related to the concept of distance from a center point? • Why is the circumference of a circle proportional to its diameter, and what role does the constant π play in this relationship? • How can we informally derive the formula for the area of a circle, and why is it equal to π times the square of its radius? • How do the relationships between circumference, radius, diameter, and area of a circle enable us to find lengths and areas in practical scenarios?
Enduring Understandings: • A circle is defined as the set of points that are equally distant from a center point, providing a geometric representation of

distance and symmetry.

- The circumference of a circle is proportional to its diameter, with π serving as the constant of proportionality, illustrating the relationship between the linear measure of the circle and its diameter.
- The area of a circle can be informally derived by considering the arrangement of concentric circles, leading to the formula $A = \pi r^2$, where r is the radius, highlighting the relationship between the area and the square of the radius.
- Understanding the relationships between circumference, radius, diameter, and area of a circle allows us to calculate lengths and areas in various contexts, expressing them in terms of π or using approximations of π to find numerical values, facilitating problem-solving and mathematical reasoning.

Key NJSLS Strands:

- **7.RP.A** Analyze proportional relationships and use them to solve real-world and mathematical problems.
 - **7.RP.A.2** Recognize and represent proportional relationships between quantities.
 - **7.RP.A.2.a** Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
 - **7.RP.A.2.c** Represent proportional relationships by equations.
- **7.G.A** Draw, construct, and describe geometrical figures and describe the relationships between them.
 - **7.G.A.2** Draw (with technology, with ruler and protractor, as well as freehand) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
- **7.G.B** Solve real-life and mathematical problems involving angle measure, area, surface area, and volume
 - **7.G.B.4** Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.
 - **7.G.B.6** Solve real-life and mathematical problems involving angle measure, area, surface area, and volume. **Climate Change Example: Students may solve real-world problems involving area, surface area, and volume related to deforestation and increasing livestock farming as key contributors to climate change.**
- **7.EE.B** Solve real-life and mathematical problems using numerical and algebraic expressions and equations
 - **7.EE.B.3** Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. **Climate Change Example: Students may solve multi-step**

real-life problems posed with positive and negative rational numbers in any form related to the relationship between altitude and the temperature above sea level.

Interdisciplinary Connections				
● NJSLSA.R7 Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words. ● NJSLSA.SL1 Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively. ● NJSLSA.SL4 Present information, findings, and supporting evidence such that listeners can follow the line of reasoning and the organization, development, and style are appropriate to task, purpose, and audience.				
21st-Century Skills				
● 9.1.8.PB.2 Explain how different circumstances can affect one's personal budget. ● 9.1.8.PB.4 Construct a simple personal savings and spending plan based on various sources of income and different stages of life (e.g. teenager, young adult, family). ● 9.1.8.PB.6 Construct a budget to save for short-term, long-term, and charitable goals.				
Career Ready Practices				
● CRP2 Apply appropriate academic and technical skills. ● CRP8 Utilize critical thinking to make sense of problems and persevere in solving them. ● CRP11 Use technology to enhance productivity.				
Technology Integration				
● 9.4.5.CT.3 Describe how digital tools and technology may be used to solve problems.				

Skills	Instructional Topics	Assessment	Resources	Vocabulary
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Decide whether two quantities show a proportional relationship by testing for equivalent ratios in a table. Decide whether two quantities show a proportional relationship by graphing on a coordinate plane and observing whether the graph is a straight line through the origin.	Circumference of a Circle Exploring Circles Exploring Circumference Applying Circumference Circumference and Wheels Area of a Circle Estimating Areas Exploring the Area of a Circle Relating Area to Circumference Applying Area of Circles Distinguishing Circumference and Area	Formative Assessments: Diagnostic Assessment Mid-topic Checkpoint Lesson Quizzes Lesson Self-assessment Teacher Observations Class Discussions Classwork/ Homework Exit Slips/Google Form Summative Assessments: Topic/Unit Assessments Constructed Response Questions Projects Cumulative Tests Performance Assessments	Materials: blank paper coins colored pencils compasses cylindrical household items empty toilet paper roll four-function calculators fruits or vegetables tracing paper graph paper colored pencils scissors index cards rulers protractors glue or glue sticks	relationship perimeter radius diameter circumference center (of a circle) circle pi half-circle rotation approximation diameter circumference pi travel approximate estimate
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		Alternate and Benchmark Assessments: Progress Monitoring Assessments Cumulative/Benchmark Assessments	graph paper grocery store circulars index cards internet-enabled device knife maps or satellite images of the school grounds markers materials assembled from the blackline master measuring cup measuring spoons measuring tapes measuring tools metal paper fasteners (brass brads) meter sticks metric and customary unit conversion charts	area of a circle squared formula radius area of a circle squared center (of a circle) formula design
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			mixing containers number cubes pain paper bags paper clips paper cups paper plates pattern blocks pre-assembled polyhedra protractors receipt tape recipes rulers rulers marked with centimeters rulers marked with inches	
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			scissors small disposable cups snap cubes sticky notes stopwatches straightedges straws string tape tools for creating a visual display trundle wheels water yardsticks Activities: String Circumference Activity Circumference	
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			Exploration Stations Circle Properties Investigation Circle Art Project Pizza Pie Activity Area Exploration Stations Guess the Area Challenge Area Estimation Activity Circle Comparison Investigation Area-Circumference Connection Task Cards Real-world Circles Problem-solving Circle Challenge Projects Mentor Texts: <i>Sir Cumference and the First Round Table</i> by Cindy Neuschwander	
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Modifications and Accommodations	
Tier II and III Strategies	Small group instruction (Tier II) ● Explicit instruction ● Schema-based instruction ● Use visuals ● Use manipulatives ● Use a multi-sensory approach ● Guided notes and/or scaffold outline for written assignments ● Introduce key vocabulary before lesson ● Lesson taught again using a differentiated approach Small group or Individual instruction (Tier III) ● Intensive instruction ● Practice fact retrieval ● Lesson taught again using a differentiated approach ● Allow answers to be given orally or dictated ● Use visuals / Anchor Charts
Students with IEPs and 504 Plans	<u>Universal Design for Learning and Access for Students with Disabilities</u> <u>Movematics (dyscalculia.org)</u> Follow all the IEP/504 plan modifications Allow extra time to complete assignments or tests Guided notes and/or scaffold outline for written assignments Work in a small group Solidify and refine concepts through repetition

	Allow answers to be given orally or dictated Use multi-sensory teaching approaches Utilize assistive technology and materials Use large print books, Braille, or books on CD (digital text) Text to speech/audio recorded selections Amplification system as needed Leveled texts according to ability Fine motor skill stations embedded in rotation as needed Modified or constrained spelling word lists Provide anchor charts with high frequency words and phonemic patterns Students might need guidance as to what actions happen during partner work time. Invite students to discuss what it looks like when they are working with their partner while exploring the connecting cubes. Generate a list of shared expectations for partner work. Invite students to plan a strategy before they begin working. If time allows, invite students to share their plan with a partner. Some students may benefit from feedback that emphasizes effort and time on task. For example, moving counters in the squares of the 5-frame or filling the 5-frame with one counter in each square.
Multi-language Learners	<u>Mathematical Language Development and Access for English Learners</u> Work toward longer passages as skills in English increase

	Use visuals Introduce/highlight key vocabulary before lesson Teacher models reading aloud daily Provide peer tutoring Utilize translation supports including bilingual dictionary, translation device, assistive technology, and peers Guided notes and/or scaffold outline for written assignments Provide students with English Learner leveled readers Support recommended by the grade specific “can do” list outlined by WIDA, including Key Uses Edition and Descriptor Name Charts Create a visual display of what students want to do or make with the connecting cubes. As students share ideas, include a drawing or annotation to illustrate connections. Invite students to use gestures as they count.
Accelerating Students	<u>Accelerating Students Guide</u> Create an enhanced set of introductory activities (e.g. advance organizers, concept maps, concept puzzles) Provide options, alternatives and choices to differentiate and broaden the curriculum Organize and offer flexible small group learning activities Provide whole group enrichment explorations

	Teach cognitive and methodological skills Use center, stations, or contracts Organize integrated problem-solving simulations Propose interest-based extension activities Expose students to beyond level texts.
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(NJDOE, 2024)

Unit 4: Proportional Relationships and Percentages

Unit Goal: In this unit, students use ratios, scale factors, unit rates (also called constants of proportionality), and proportional relationships to solve multi-step, real-world problems that involve fractions and percentages. They use long division to write fractions presented in the form $\frac{a}{b}$ as decimals, e.g., $\frac{11}{30} = 0.3\overline{6}$. They learn to understand and use the terms “repeating decimal,” “terminating decimal,” “percent increase,” “percent decrease,” “percent error,” and “measurement error.” They represent amounts and corresponding percent rates with double number line diagrams and tables. They use these terms and representations in reasoning about situations involving sales taxes, tips, markdowns, markups, sales commissions, interest, depreciation, and scaling a picture. Students use equations to represent proportional relationships in which the constant of proportionality arises from a percentage, e.g., relationship between price paid and amount of sales tax paid.

Duration: 4 weeks

Essential Questions:

- How do ratios, scale factors, and unit rates help us solve real-world problems involving fractions and percentages?
- What are the characteristics of repeating and terminating decimals, and how do they relate to fractions?
- How do percent increase and percent decrease quantify changes in quantities, and how are they calculated?
- How do we use terms like “percent error” and “measurement error” to assess the accuracy of calculations and measurements?
- How can double number line diagrams and tables be used to represent amounts and corresponding percent rates in proportional relationships?
- How do sales taxes, tips, markdowns, markups, and other financial transactions involve proportional relationships and percentages?

- How can equations be used to represent proportional relationships where the constant of proportionality arises from percentages?

Enduring Understandings:

- Ratios, scale factors, and unit rates are powerful tools for solving real-world problems involving fractions and percentages, allowing us to make comparisons and calculate quantities efficiently.
- Repeating and terminating decimals represent different types of rational numbers, and understanding their properties helps us convert between fractions and decimals accurately.
- Percent increase and percent decrease are useful measures for quantifying changes in quantities relative to their original values, providing insights into trends and fluctuations.
- Percent error and measurement error are important concepts for evaluating the accuracy of calculations and measurements, informing decisions and problem-solving strategies.
- Double number line diagrams and tables provide visual representations of proportional relationships between amounts and corresponding percent rates, aiding in comprehension and problem-solving.
- Financial transactions such as sales taxes, tips, and markups involve proportional relationships and percentages, requiring mathematical reasoning and problem-solving skills to navigate effectively.
- Equations can represent proportional relationships where the constant of proportionality arises from percentages, providing a mathematical framework for analyzing and solving problems involving proportional reasoning and percentages.

Key NJSLS Strands:

- **7.RP.A** Analyze proportional relationships and use them to solve real-world and mathematical problems
 - **7.RP.A.1** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.
 - **7.RP.A.2** Recognize and represent proportional relationships between quantities.
 - **7.RP.A.2.a** Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
 - **7.RP.A.3** Use proportional relationships to solve multistep ratio and percent problems.
- **7.EE.A** Use properties of operations to generate equivalent expressions.
 - **7.EE.A.1** Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

- **7.NS.A** Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.
 - **7.NS.A.2** Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.
 - **7.NS.A.2.d** Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

Interdisciplinary Connections

- **NJSLSA.R7** Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words.
- **NJSLSA.SL1** Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively.
- **NJSLSA.SL4** Present information, findings, and supporting evidence such that listeners can follow the line of reasoning and the organization, development, and style are appropriate to task, purpose, and audience.
- **MS-ETS1-1** Students estimate sums and differences while defining the criteria and constraints of a design problem with sufficient precision to ensure a successful solution, taking into account relevant scientific principles.

21st-Century Skills

- **9.1.8.EG.1:** Explain how taxes affect disposable income and the difference between net and gross income
- **9.1.8.EG.2:** Explain why various sources of income are taxed differently.
- **9.1.8.EG.3:** Explain the concept and forms of taxation and evaluate how local, state and federal governments use taxes to fund public activities and initiatives.
- **9.1.8.EG.4:** Identify and explain the consequences of breaking federal and/or state employment or financial laws.

Career Ready Practices

- **CRP2** Apply appropriate academic and technical skills.

Technology Integration

- **9.4.5.CT.3** Describe how digital tools and technology may be used to solve problems.

Skills	Instructional Topics	Assessment	Resources	Vocabulary
Compute unit rates involving ratios of fractions (complex fractions) in quantities measured in like or different units. Decide whether two quantities show a proportional relationship by testing for equivalent ratios in a table. Decide whether two quantities show a proportional relationship by graphing on a coordinate plane and observing whether the graph is a straight line through the origin. Convert a rational number to a decimal using long division.	Proportional Relationships with Fractions Ratios and Rates With Fractions Revisiting Proportional Relationships Half as Much Say It with Decimals Percent Increase and Decrease Increasing and Decreasing One Hundred Percent Percent Increase and Decrease with Equations More and Less than 1%	Formative Assessments: Diagnostic Assessment Mid-topic Checkpoint Lesson Quizzes Lesson Self-assessment Teacher Observations Class Discussions Classwork/ Homework Exit Slips/Google Form Summative Assessments: Topic/Unit Assessments Constructed Response Questions Projects	Materials: blank paper coins colored pencils compasses cylindrical household items empty toilet paper roll four-function calculators fruits or vegetables tracing paper graph paper colored pencils	percentage unit rate (a fraction) more than (a fraction) less than initial / original amount final / new amount tape diagram distributive property repeating decimal long division decimal representation percent increase percent decrease (a fraction) more than

Explain that the decimal form of a rational number terminates in zeros or eventually repeats. Apply the properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.	Applying Percentages Tax and Tip Percentage Contexts Finding the Percentage Measurement Error Percent Error Error Intervals Posing Percentage Problems	Cumulative Tests Performance Assessments Alternate and Benchmark Assessments: Progress Monitoring Assessments Cumulative/Benchmark Assessments	scissors index cards rulers protractors glue or glue sticks graph paper grocery store circulars index cards internet-enabled device knife maps or satellite images of the school grounds markers materials assembled from the blackline master measuring cup measuring spoons	(a fraction) less than discount initial / original amount final / new amount sales tax tax rate tip percent increase interest commission markup markdown percent decrease percentage discount measurement error percent error
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			measuring tapes measuring tools metal paper fasteners (brass brads) meter sticks metric and customary unit conversion charts mixing containers number cubes pain paper bags paper clips paper cups paper plates pattern blocks pre-assembled polyhedra protractors	temperature degrees Fahrenheit
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			receipt tape recipes rulers rulers marked with centimeters rulers marked with inches scissors small disposable cups snap cubes sticky notes stopwatches straightedges straws string tape tools for creating a visual display	
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			trundle wheels water yardsticks Activities: Fractional Scaling Task Fractional Comparison Challenge Fractional Ratio Exploration Rate Conversion Practice Proportional Relationship Review Stations Proportional Relationship Reflection Percent Change Task Cards Real-world Percent Change Problems Tax and Tip Simulation	
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			Percentage Contexts Project Measurement Error Investigation Error Intervals Activity Mentor Texts: <i>The Taxing Case of the Cows: A True Story</i> <i>About Suffrage</i> by Iris Van Rynbach	
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Modifications and Accommodations	
Tier II and III Strategies	Small group instruction (Tier II) ● Explicit instruction ● Schema-based instruction ● Use visuals ● Use manipulatives ● Use a multi-sensory approach ● Guided notes and/or scaffold outline for written assignments ● Introduce key vocabulary before lesson ● Lesson taught again using a differentiated approach Small group or Individual instruction (Tier III) ● Intensive instruction ● Practice fact retrieval ● Lesson taught again using a differentiated approach ● Allow answers to be given orally or dictated

	● Use visuals / Anchor Charts
Students with IEPs and 504 Plans	<u>Universal Design for Learning and Access for Students with Disabilities</u> <u>Movematics (dyscalculia.org)</u> Follow all the IEP/504 plan modifications Allow extra time to complete assignments or tests Guided notes and/or scaffold outline for written assignments Work in a small group Solidify and refine concepts through repetition Allow answers to be given orally or dictated Use multi-sensory teaching approaches Utilize assistive technology and materials Use large print books, Braille, or books on CD (digital text) Text to speech/audio recorded selections Amplification system as needed Leveled texts according to ability Fine motor skill stations embedded in rotation as needed Modified or constrained spelling word lists Provide anchor charts with high frequency words and phonemic patterns

	Students might need guidance as to what actions happen during partner work time. Invite students to discuss what it looks like when they are working with their partner while exploring the connecting cubes. Generate a list of shared expectations for partner work. Invite students to plan a strategy before they begin working. If time allows, invite students to share their plan with a partner. Some students may benefit from feedback that emphasizes effort and time on task. For example, moving counters in the squares of the 5-frame or filling the 5-frame with one counter in each square.
Multi-language Learners	<u>Mathematical Language Development and Access for English Learners</u> Work toward longer passages as skills in English increase Use visuals Introduce/highlight key vocabulary before lesson Teacher models reading aloud daily Provide peer tutoring Utilize translation supports including bilingual dictionary, translation device, assistive technology, and peers Guided notes and/or scaffold outline for written assignments Provide students with English Learner leveled readers Support recommended by the grade specific “can do” list outlined by WIDA, including Key Uses Edition and Descriptor Name Charts Create a visual display of what students want to do or make with the connecting cubes. As students share ideas, include a drawing or annotation to illustrate connections.

	Invite students to use gestures as they count.
Accelerating Students	<u>Accelerating Students Guide</u> Create an enhanced set of introductory activities (e.g. advance organizers, concept maps, concept puzzles) Provide options, alternatives and choices to differentiate and broaden the curriculum Organize and offer flexible small group learning activities Provide whole group enrichment explorations Teach cognitive and methodological skills Use center, stations, or contracts Organize integrated problem-solving simulations Propose interest-based extension activities Expose students to beyond level texts.

(NJDOE, 2024)

Unit 5: Rational Number Arithmetic

Unit Goal: In this unit, students interpret signed numbers in contexts (e.g., temperature, elevation, deposit and withdrawal, position, direction, speed and velocity, percent change) together with their sums, differences, products, and quotients. (“Signed numbers” include all rational numbers, written as decimals or in the form $\frac{a}{b}$.) Students use tables and number line diagrams to represent sums and differences of signed numbers or changes in quantities represented by signed numbers such as temperature or elevation, becoming more fluent in writing different numerical addition and subtraction equations that express the same relationship. They

compute sums and differences of signed numbers. They plot points in the plane with signed number coordinates, representing and interpreting sums and differences of coordinates. They view situations in which objects are traveling at constant speed (familiar from previous units) as proportional relationships. For these situations, students use multiplication equations to represent changes in position on number line diagrams or distance traveled, and interpret positive and negative velocities in context. They become more fluent in writing different numerical multiplication and division equations for the same relationship. Students extend their use of the “next to” notation to include negative numbers and products of numbers.

Duration: 3.5 weeks

Essential Questions:

- How do signed numbers represent quantities in different real-world contexts, and how do we interpret their meanings in relation to positive and negative values?
- In what ways do tables and number line diagrams help us visualize and understand the relationships between signed numbers, including their sums, differences, products, and quotients?
- How can we use coordinate planes to represent points with signed number coordinates, and what geometric interpretations can we derive from these representations?
- What are the characteristics of proportional relationships in constant speed situations, and how do multiplication equations help us model changes in position or distance traveled?
- How does fluency in writing different numerical equations for the same relationship enhance our ability to express and manipulate mathematical concepts involving signed numbers?
- How does the inclusion of negative numbers and products of numbers extend the use of mathematical notation and notation for relationships like "next to"?

Enduring Understandings:

- Signed numbers, including positive and negative rational numbers, are used to represent quantities in various contexts such as temperature, elevation, financial transactions, and spatial relationships.
- Number line diagrams and tables provide visual representations for interpreting and manipulating sums, differences, products, and quotients of signed numbers, enhancing fluency in mathematical operations.
- Understanding of signed numbers allows students to plot points with signed number coordinates on the coordinate plane and interpret geometric relationships between these points.
- Constant speed situations can be modeled using proportional relationships, where multiplication equations represent changes in position or distance traveled, enabling interpretation of positive and negative velocities in context.
- Fluency in writing different numerical equations for the same relationship, including addition, subtraction, multiplication,

and division, enhances students' ability to express mathematical relationships in various forms.

Key NJSL Strands:

- **7.NS.A** Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers
 - **7.NS.A.1** Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
 - **7.NS.A.1.b** Understand $p + q$ as the number located a distance $|q|$ from p , in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
 - **7.NS.A.1.c** Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
 - **7.NS.A.2** Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.
 - **7.NS.A.2.a** Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
 - **7.NS.A.2.b** Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If q and q are integers, then $-\left(\frac{p}{q}\right) = \frac{(-p)}{q} = \frac{p}{(-q)}$. Interpret quotients of rational numbers by describing real world contexts.
 - **7.NS.A.2.c** Apply properties of operations as strategies to multiply and divide rational numbers.
 - **7.NS.A.2.d** Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.
 - **7.NS.A.3** Solve real-world and mathematical problems involving the four operations with rational numbers.
- **7.EE.B** Solve real-life and mathematical problems using numerical and algebraic expressions and equations

- **7.EE.B.3** Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.
- **7.EE.B.4** Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
 - **7.EE.B.4.a** Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p , q , and r are specific rational numbers. Solve equations of these forms with accuracy and efficiency. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.

Interdisciplinary Connections

- **NJSLSA.R7** Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words.
- **NJSLSA.SL1** Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively.
- **NJSLSA.SL4** Present information, findings, and supporting evidence such that listeners can follow the line of reasoning and the organization, development, and style are appropriate to task, purpose, and audience.
- **MS-ETS1-1** Students estimate sums and differences while defining the criteria and constraints of a design problem with sufficient precision to ensure a successful solution, taking into account relevant scientific principles.

21st-Century Skills

- **9.2.8.CAP.3** Explain how career choices, educational choices, skills, economic conditions, and personal behavior affect income.

Career Ready Practices

- **CRP4** Communicate clearly and effectively with reason.

Technology Integration
● 9.4.8.TL.6 Collaborate to develop and publish work that provides perspectives on a real-world problem.

Skills	Instructional Topics	Assessment	Resources	Vocabulary
Explain that $p + q$ is the number located a distance q from p, in the positive or negative direction depending on whether q is positive or negative (e.g. $5 + -4$ is 4 units in the negative direction from 5 and, similarly, $5 + 4$ is also 4 units away in the positive direction). Represent addition and subtraction of signed rational numbers on a vertical or horizontal number line. Interpret sums of rational numbers in real world situations.	Interpreting Negative Numbers Adding and Subtracting Rational Numbers Changing Temperatures Changing Elevation Money and Debts Representing Subtraction Subtracting Rational Numbers Adding and Subtracting to Solve Problems Multiplying and Dividing Rational Numbers	Formative Assessments: Diagnostic Assessment Mid-topic Checkpoint Lesson Quizzes Lesson Self-assessment Teacher Observations Class Discussions Classwork/ Homework Exit Slips/Google Form Summative Assessments: Topic/Unit Assessments	Materials: blank paper coins colored pencils compasses cylindrical household items empty toilet paper roll four-function calculators fruits or vegetables tracing paper graph paper	absolute value degrees Celsius vertical elevation sea level positive number negative number signed numbers temperature number line sum opposite

Apply previous understandings of subtraction to subtract rational numbers. Explain that subtraction of rational numbers is the same as adding the additive inverse, $p - q = p + (-q)$. Show by modeling on a number line that the distance between two rational numbers is the absolute value of their differences and apply the concept in real world contexts. Apply previous understandings of multiplication of fractions to multiply signed rational numbers. Operations on signed rational numbers. Continue to satisfy the properties of operations, The distributive property, leading to	Position, Speed, and Direction Multiplying Rational Numbers Dividing Rational Numbers Negative Rates Four Operations with Rational Numbers Expressions with Rational Numbers Solving Problems with Rational Numbers Solving Equations When There Are Negative Numbers Solving Equations with Rational Numbers Representing Contexts with Equations	Constructed Response Questions Projects Cumulative Tests Performance Assessments Alternate and Benchmark Assessments: Progress Monitoring Assessments Cumulative/Benchmark Assessments	colored pencils scissors index cards rulers protractors glue or glue sticks graph paper grocery store circulars index cards internet-enabled device knife maps or satellite images of the school grounds markers materials assembled from the blackline master measuring cup	expression deposit withdrawal account balance debt difference distance absolute value x-coordinate y-coordinate velocity solution (to an equation) factor rational number sum difference variable
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products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret the products of signed rational numbers in real world situations. Apply previous understandings of division of fractions to divide signed rational numbers. Integers can be divided as long as the divisor is not zero. Division of integers results in a signed rational number. Explain if p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of signed rational numbers by describing real world contexts. Apply properties of operations as strategies			measuring spoons measuring tapes measuring tools metal paper fasteners (brass brads) meter sticks metric and customary unit conversion charts mixing containers number cubes pain paper bags paper clips paper cups paper plates pattern blocks pre-assembled polyhedra	additive inverse multiplicative inverse opposite solution (to an equation) operation increase decrease
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to multiply and divide signed rational numbers. Convert a rational number to a decimal using long division. The decimal form of a rational number terminates in zeros or eventually repeats.			protractors receipt tape recipes rulers rulers marked with centimeters rulers marked with inches scissors small disposable cups snap cubes sticky notes stopwatches straightedges straws string tape	
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			tools for creating a visual display trundle wheels water yardsticks Activities: Signed Numbers Exploration Number Line Fluency Coordinate Plane Plotting Constant Speed Modeling Equation Writing Fluency Extension: "Next To" Notation Real-World Context Scenarios Interactive Number Line Game	
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			Graphical Representation Task Collaborative Problem-Solving Error Analysis Peer Teaching Presentations Mentor Texts: <i>The Math Inspectors</i> Series by Daniel Kenney	
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Modifications and Accommodations	
Tier II and III Strategies	Small group instruction (Tier II) ● Explicit instruction ● Schema-based instruction ● Use visuals ● Use manipulatives ● Use a multi-sensory approach ● Guided notes and/or scaffold outline for written assignments ● Introduce key vocabulary before lesson ● Lesson taught again using a differentiated approach Small group or Individual instruction (Tier III) ● Intensive instruction ● Practice fact retrieval ● Lesson taught again using a differentiated approach ● Allow answers to be given orally or dictated

	● Use visuals / Anchor Charts
Students with IEPs and 504 Plans	<u>Universal Design for Learning and Access for Students with Disabilities</u> <u>Movematics (dyscalculia.org)</u> Follow all the IEP/504 plan modifications Allow extra time to complete assignments or tests Guided notes and/or scaffold outline for written assignments Work in a small group Solidify and refine concepts through repetition Allow answers to be given orally or dictated Use multi-sensory teaching approaches Utilize assistive technology and materials Use large print books, Braille, or books on CD (digital text) Text to speech/audio recorded selections Amplification system as needed Leveled texts according to ability Fine motor skill stations embedded in rotation as needed Modified or constrained spelling word lists Provide anchor charts with high frequency words and phonemic patterns

	Students might need guidance as to what actions happen during partner work time. Invite students to discuss what it looks like when they are working with their partner while exploring the connecting cubes. Generate a list of shared expectations for partner work. Invite students to plan a strategy before they begin working. If time allows, invite students to share their plan with a partner. Some students may benefit from feedback that emphasizes effort and time on task. For example, moving counters in the squares of the 5-frame or filling the 5-frame with one counter in each square.
Multi-language Learners	<u>Mathematical Language Development and Access for English Learners</u> Work toward longer passages as skills in English increase Use visuals Introduce/highlight key vocabulary before lesson Teacher models reading aloud daily Provide peer tutoring Utilize translation supports including bilingual dictionary, translation device, assistive technology, and peers Guided notes and/or scaffold outline for written assignments Provide students with English Learner leveled readers Support recommended by the grade specific “can do” list outlined by WIDA, including Key Uses Edition and Descriptor Name Charts Create a visual display of what students want to do or make with the connecting cubes. As students share ideas, include a drawing or annotation to illustrate connections.

	Invite students to use gestures as they count.
Accelerating Students	<u>Accelerating Students Guide</u> Create an enhanced set of introductory activities (e.g. advance organizers, concept maps, concept puzzles) Provide options, alternatives and choices to differentiate and broaden the curriculum Organize and offer flexible small group learning activities Provide whole group enrichment explorations Teach cognitive and methodological skills Use center, stations, or contracts Organize integrated problem-solving simulations Propose interest-based extension activities Expose students to beyond level texts.

(NJDOE, 2024)

Unit 6: Expressions, Equations, and Inequalities

Unit Goal: In this unit, students solve equations of the forms $px + q = r$ and $p(x+q) = r$ where p , q , and r are rational numbers. They draw, interpret, and write equations in one variable for balanced “hanger diagrams,” and write expressions for sequences of instructions, e.g., “number puzzles.” They use tape diagrams together with equations to represent situations with one unknown quantity. They learn algebraic methods for solving equations. Students solve linear inequalities in one variable and represent their solutions on the number line. They understand and use the terms “less than or equal to” and “greater than or equal to,” and the

corresponding symbols. They generate expressions that are equivalent to a given numerical or linear expression. Students formulate and solve linear equations and inequalities that represent real-world situations.

Duration: 5 weeks

Essential Questions:

- How can we use algebraic methods to solve equations in one variable, and what strategies can we employ to find solutions efficiently?
- What are hanger diagrams and number puzzles, and how do they help us write and interpret equations in one variable?
- How do tape diagrams assist us in representing real-world situations involving one unknown quantity, and how do we translate verbal descriptions into mathematical expressions and equations?
- **What is the significance of linear inequalities in one variable, and how do we represent their solutions on the number line using symbols such as $\leq$ and $\geq$?**
- Why is it important to recognize equivalent expressions, and how can we generate expressions that maintain the same value as a given numerical or linear expression?
- How do we apply the principles of linear equations and inequalities to formulate and solve mathematical models for real-world situations, and what insights do these solutions provide?

Enduring Understandings:

- Equations in one variable, including linear equations of the forms $px + q = r$ and $p(x+q) = r$, can be solved using algebraic methods, leading to the discovery of solutions that satisfy the given conditions.
- Hanger diagrams and number puzzles provide visual representations for writing and interpreting equations in one variable, enhancing understanding of the relationships between variables and constants.
- Tape diagrams serve as powerful tools for representing and solving real-world problems involving one unknown quantity, facilitating the translation of verbal descriptions into mathematical expressions and equations.
- **Linear inequalities in one variable represent relationships where one quantity is greater than or less than another, with solutions often depicted on the number line using symbols such as $\leq$ and $\geq$.**
- Equivalent expressions maintain the same value despite differences in their numerical or linear forms, allowing for simplification and manipulation of mathematical expressions.
- Formulating and solving linear equations and inequalities enable students to address and solve real-world problems in

various contexts, applying mathematical principles to analyze and interpret situations.

Key NJSLS Strands:

- **7.EE.B** Solve real-life and mathematical problems using numerical and algebraic expressions and equations.
 - **7.EE.B.3** Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.
 - **7.EE.B.4** Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
 - **7.EE.B.4.a** Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p , q , and r are specific rational numbers. Solve equations of these forms with accuracy and efficiency. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.
 - **7.EE.B.4.b** Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p , q , and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.
- **7.NS.A** Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers
 - **7.NS.A.1** Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
 - **7.NS.A.1.c** Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

Interdisciplinary Connections

- **NJSLSA.R7** Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words.

● NJSLSA.SL1 Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively ● NJSLSA.SL4 Present information, findings, and supporting evidence such that listeners can follow the line of reasoning and the organization, development, and style are appropriate to task, purpose, and audience. ● MS-ETS1-1 Students estimate sums and differences while defining the criteria and constraints of a design problem with sufficient precision to ensure a successful solution, taking into account relevant scientific principles.
21st-Century Skills
● 9.4.8.CT.2 Develop multiple solutions to a problem and evaluate short- and long-term effects to determine the most plausible option).
Career Ready Practices
● CRP2 Apply appropriate academic and technical skills.
Technology Integration
● 9.4.5.CT.3 Describe how digital tools and technology may be used to solve problems.

Skills	Instructional Topics	Assessment	Resources	Vocabulary
Convert between forms (fractions, decimals, and whole numbers) as appropriate to solve multi-step real life and mathematical problems with positive and negative rational numbers in any form.	Representing Situations of the Form $px + q = r$ and $p(x+q) = r$ Relationships between Quantities	Formative Assessments: Diagnostic Assessment Mid-topic Checkpoint Lesson Quizzes Lesson Self-assessment	Materials: blank paper coins colored pencils compasses	unknown amount equivalent expressions commutative (property) unknown amount relationship

Apply the properties of operations to calculate with numbers in any form when solving multi-step real-life and mathematical problems, and assess the reasonableness of answers using mental computation and estimation strategies. Solve world problems by reasoning about their quantities and constructing simple equations of the form $p(x + q) = r$, where p, q, and r are specific rational numbers. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. Solve equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers with accuracy	Reasoning about Contexts with Tape Diagrams Reasoning about Equations with Tape Diagrams Reasoning about Equations and Tape Diagrams Distinguishing between Two Types of Situations Solving Equations of the Form and Problems That Lead to Those Equations Solving Equations of the Form $px + q = r$ and $p(x + q) = r$ And Problems That Lead to Those Equations Reasoning about Solving Equations	Teacher Observations Class Discussions Classwork/ Homework Exit Slips/Google Form Summative Assessments: Topic/Unit Assessments Constructed Response Questions Projects Cumulative Tests Performance Assessments Alternate and Benchmark Assessments: Progress Monitoring Assessments Cumulative/Benchmark Assessments	cylindrical household items empty toilet paper roll four-function calculators fruits or vegetables tracing paper graph paper colored pencils scissors index cards rulers protractors glue or glue sticks graph paper grocery store circulars index cards internet-enabled device	variable balanced hanger each side (of an equation) equivalent expression each side (of an equation) operation solve distribute substitute inequality less than or equal to greater than or equal to open / closed circle less than greater than solution to an inequality
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and efficiency. Apply previous understandings of addition to add rational numbers. Describe situations in which opposites combine to make zero. Show by modeling, a number and its opposite have a sum of zero (additive inverse). Apply previous understandings of subtraction to subtract rational numbers. Subtraction of rational numbers is the same as adding the additive inverse, $p - q = p + (-q)$. Show by modeling on a number line that the distance between two rational numbers is the absolute value of their	Dealing with Negative Numbers Different Options for Solving One Equation Using Equations to Solve Problems Solving Problems about Percent Increase or Decrease Inequalities Inequalities Reintroducing Inequalities Finding Solutions to Inequalities in Context Efficiently Solving Inequalities Interpreting Inequalities Modeling with Inequalities		knife maps or satellite images of the school grounds markers materials assembled from the blackline master measuring cup measuring spoons measuring tapes measuring tools metal paper fasteners (brass brads) meter sticks metric and customary unit conversion charts mixing containers number cubes pain	boundary direction (of an inequality) less than or equal to greater than or equal to substitute open / closed circle solution to an inequality inequality term factor (an expression) expand (an expression) combine like terms term commutative (property) distribute associative property factor (an expression)
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differences and apply the concept in real world contexts.	Writing Equivalent Expressions Subtraction in Equivalent Expressions Expanding and Factoring Combining Like Terms		paper bags paper clips paper cups paper plates pattern blocks pre-assembled polyhedra protractors receipt tape recipes rulers rulers marked with centimeters rulers marked with inches scissors small disposable cups snap cubes	expand (an expression)
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			sticky notes stopwatches straightedges straws string tape tools for creating a visual display trundle wheels water yardsticks Activities: Equation Solving Relay Hanger Diagram Puzzles Number Puzzle Expressions Tape Diagram Challenges	
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			Inequality Exploration Equivalent Expression Matching Real-World Equation Formulation Algebraic Puzzle Games Interactive Equation Balancing Mathematical Modeling Projects Mentor Texts: <i>The Secret of the Old Clock</i> by Carolyn Keene (This mystery novel involves solving puzzles and deciphering clues, providing inspiration for creating algebraic puzzle games that engage students in problem-solving.)	
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Modifications and Accommodations	
Tier II and III Strategies	Small group instruction (Tier II) • Explicit instruction • Schema-based instruction

	● Use visuals ● Use manipulatives ● Use a multi-sensory approach ● Guided notes and/or scaffold outline for written assignments ● Introduce key vocabulary before lesson ● Lesson taught again using a differentiated approach Small group or Individual instruction (Tier III) ● Intensive instruction ● Practice fact retrieval ● Lesson taught again using a differentiated approach ● Allow answers to be given orally or dictated ● Use visuals / Anchor Charts
Students with IEPs and 504 Plans	<u>Universal Design for Learning and Access for Students with Disabilities</u> <u>Movematics (dyscalculia.org)</u> Follow all the IEP/504 plan modifications Allow extra time to complete assignments or tests Guided notes and/or scaffold outline for written assignments Work in a small group Solidify and refine concepts through repetition Allow answers to be given orally or dictated Use multi-sensory teaching approaches Utilize assistive technology and materials Use large print books, Braille, or books on CD (digital text)

	Text to speech/audio recorded selections Amplification system as needed Leveled texts according to ability Fine motor skill stations embedded in rotation as needed Modified or constrained spelling word lists Provide anchor charts with high frequency words and phonemic patterns Students might need guidance as to what actions happen during partner work time. Invite students to discuss what it looks like when they are working with their partner while exploring the connecting cubes. Generate a list of shared expectations for partner work. Invite students to plan a strategy before they begin working. If time allows, invite students to share their plan with a partner. Some students may benefit from feedback that emphasizes effort and time on task. For example, moving counters in the squares of the 5-frame or filling the 5-frame with one counter in each square.
Multi-language Learners	<u>Mathematical Language Development and Access for English Learners</u> Work toward longer passages as skills in English increase Use visuals Introduce/highlight key vocabulary before lesson Teacher models reading aloud daily Provide peer tutoring Utilize translation supports including bilingual dictionary, translation device, assistive

	technology, and peers Guided notes and/or scaffold outline for written assignments Provide students with English Learner leveled readers Support recommended by the grade specific “can do” list outlined by WIDA, including Key Uses Edition and Descriptor Name Charts Create a visual display of what students want to do or make with the connecting cubes. As students share ideas, include a drawing or annotation to illustrate connections. Invite students to use gestures as they count.
Accelerating Students	<u>Accelerating Students Guide</u> Create an enhanced set of introductory activities (e.g. advance organizers, concept maps, concept puzzles) Provide options, alternatives and choices to differentiate and broaden the curriculum Organize and offer flexible small group learning activities Provide whole group enrichment explorations Teach cognitive and methodological skills Use center, stations, or contracts Organize integrated problem-solving simulations Propose interest-based extension activities Expose students to beyond level texts.

Unit 7: Angles, Triangles, and Prisms

Unit Goal: In this unit, students investigate whether sets of angle and side length measurements determine unique triangles or multiple triangles, or fail to determine triangles. Students also study and apply angle relationships, learning to understand and use the terms “complementary,” “supplementary,” “vertical angles,” and “unique.” The work gives them practice working with rational numbers and equations for angle relationships. Students analyze and describe cross-sections of prisms, pyramids, and polyhedra. They understand and use the formula for the volume of a right rectangular prism, and solve problems involving area, surface area, and volume.

Duration: 4 weeks

Essential Questions:

- How do sets of angle and side length measurements determine the uniqueness or multiplicity of triangles, and what factors contribute to these determinations?
- What are complementary, supplementary, and vertical angles, and how do these angle relationships facilitate the analysis of geometric shapes and properties?
- How are rational numbers and equations used to represent and solve angle relationships, and what advantages do these mathematical tools offer in geometric analysis?
- What insights can be gained from analyzing cross-sections of prisms, pyramids, and polyhedra, and how do these insights contribute to our understanding of spatial relationships?
- Why is the formula for the volume of a right rectangular prism important, and how can it be applied to solve problems involving area, surface area, and volume in real-world contexts?

Enduring Understandings:

- Sets of angle and side length measurements can determine unique triangles, multiple triangles, or fail to determine triangles altogether, depending on the relationships between the measurements.
- Angle relationships, including complementary, supplementary, and vertical angles, provide insights into the geometric properties of shapes and facilitate the analysis of angles within polygons.
- Rational numbers and equations are used to represent and solve angle relationships, allowing for precise mathematical descriptions of geometric properties.
- Cross-sections of prisms, pyramids, and polyhedra reveal geometric shapes formed by intersecting planes, providing insights

into the spatial relationships between these shapes.

- The formula for the volume of a right rectangular prism enables students to calculate the amount of space enclosed by three-dimensional objects, while problems involving area, surface area, and volume foster deeper understanding of geometric concepts and their applications.

Key NJSLS Strands:

- **7.G.A** Draw, construct, and describe geometrical figures and describe the relationships between them.
 - **7.G.A.2** Draw (with technology, with ruler and protractor, as well as freehand) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
 - **7.G.A.3** Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.
- **7.G.B** Solve real-life and mathematical problems involving angle measure, area, surface area, and volume
 - **7.G.B.5** Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.
 - **7.G.B.6** Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.
- **7.EE.A** Use properties of operations to generate equivalent expressions.
- **7.EE.B** Solve real-life and mathematical problems using numerical and algebraic expressions and equations.
 - **7.EE.B.4** Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
- **7.NS.A** Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers
 - **7.NS.A.1** Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
- **7.RP.A** Analyze proportional relationships and use them to solve real-world and mathematical problems.

Interdisciplinary Connections

- **ELA: W.6.1** When students are justifying their reasoning on short answer and extended constructed response questions they write arguments to support claims with clear reasons and relevant evidence.

● MS-ESS1-3 When students are creating and comparing ratios they analyze and interpret data to determine scale properties of objects in the solar system
21st-Century Skills
● 9.2.8.CAP.12 Assess personal strengths, talents, values, and interests to appropriate jobs and careers to maximize career potential.
Career Ready Practices
● CRP2 Apply appropriate academic and technical skills. ● CRP8 Utilize critical thinking to make sense of problems and persevere in solving them. ● CRP11 Use technology to enhance productivity.
Technology Integration
● 9.4.8.TL.3 Select appropriate tools to organize and present information digitally. ● 9.4.5.CT.3 Describe how digital tools and technology may be used to solve problems.

Skills	Instructional Topics	Assessment	Resources	Vocabulary
Solve real-world and mathematical problems involving areas of two-dimensional objects composed of triangles, quadrilaterals, and polygons. Understand supplementary angles are two angles whose sum is 180 degrees and	Angle Relationships Relationships of Angles Adjacent Angles Nonadjacent Angles Solving for Unknown Angles Using Equations to	Formative Assessments: Diagnostic Assessment Mid-topic Checkpoint Lesson Quizzes Lesson Self-assessment Teacher Observations	Materials: blank paper coins colored pencils compasses	straight angle adjacent angles degree right angle supplementary complementary

complementary angles are two angles whose sum is 90 degrees. Explain that vertical angles, the pairs of opposite angles made by two intersecting lines, have equal measures. Explain that adjacent angles are two angles that share a vertex and a side. Use facts about supplementary, complementary, vertical and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. Draw geometric shapes with given conditions with technology, with rulers and protractors, as well as freehand. Construct triangles from three measures of	Solve for Unknown Angles Drawing Polygons with Given Conditions Building Polygons Triangles with 3 Common Measures Drawing Triangles Solid Geometry Slicing Solids Volume of Right Prisms Decomposing Bases for Area Surface Area of Right Prisms Distinguishing Volume and Surface Area Applying Volume and Surface Area Building Prisms	Class Discussions Classwork/ Homework Exit Slips/Google Form Summative Assessments: Topic/Unit Assessments Constructed Response Questions Projects Cumulative Tests Performance Assessments Alternate and Benchmark Assessments: Progress Monitoring Assessments Cumulative/Benchmark Assessments	cylindrical household items empty toilet paper roll four-function calculators fruits or vegetables tracing paper graph paper colored pencils scissors index cards rulers protractors glue or glue sticks graph paper grocery store circulars index cards internet-enabled device	angle measure protractor measurement error degrees vertical angles intersect vertex (of an angle) supplementary vertical angles perpendicular complementary identical copy condition angle measure side length quadrilateral compass
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angles or sides using technology and notice when the conditions determine a unique triangle, more than one triangle, or no triangle. Construct triangles from three measures of angles or sides using rulers and protractors and notice when the conditions determine a unique triangle, more than one triangle, or no triangle. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. Solve real-world and mathematical problems involving volume and surface area of three-dimensional objects composed of			knife maps or satellite images of the school grounds markers materials assembled from the blackline master measuring cup measuring spoons measuring tapes measuring tools metal paper fasteners (brass brads) meter sticks metric and customary unit conversion charts mixing containers number cubes pain	different triangle intersect identical copy segment condition different triangle unique triangle parallel protractor compass cross section base (of a prism or pyramid) vertex (of a pyramid) face prism pyramid perpendicular
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cubes and right prisms. Solve world problems by reasoning about their quantities and constructing simple equations of the form $p(x + q) = r$, where p, q, and r are specific rational numbers. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. Solve equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers with accuracy at rational numbers. Subtraction of rational numbers is the same as adding the additive inverse, $p - q = p + (-q)$. Show by modeling on a			paper bags paper clips paper cups paper plates pattern blocks pre-assembled polyhedra protractors receipt tape recipes rulers rulers marked with centimeters rulers marked with inches scissors small disposable cups snap cubes	parallel volume cross section base (of a prism or pyramid) face perimeter surface area
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number line the distance between and efficiency. Use variables to represent unknown quantities in mathematical problems to construct and solve simple inequalities. Describe the solution of an inequality using a graph and inequality statement and interpret its meaning in the context of the problem. Compute unit rates involving ratios of fractions (complex fractions) in quantities measured in like or different unit. Decide whether two quantities show a proportional relationship by testing for equivalent ratios in a table. Decide whether two quantities show a proportional			sticky notes stopwatches straightedges straws string tape tools for creating a visual display trundle wheels water yardsticks Activities: Polygon Area Problem Solving Angle Sum Understanding Angle Relationship Explanation	
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relationship by graphing on a coordinate plane and observing whether the graph is a straight line through the origin. Identify the constant of proportionality (unit rate) in equations and verbal descriptions of proportional relationships. Identify the constant of proportionality (unit rate) in tables, graphs, and diagrams. Explain that $p + q$ is the number located a distance q from p, in the positive or negative direction depending on whether q is positive or negative (e.g. $5 + -4$ is 4 units in the negative direction from 5 and, similarly, $5 + 4$ is also 4 units away in the positive direction). Represent addition and subtraction of signed rational numbers on a			Multi-step Angle Problems Geometric Shape Drawing Triangle Construction with Technology Triangle Construction with Tools 3D Figure Plane Sections Description Volume and Surface Area Problem Solving Rational Equation Problem Solving Algebraic vs. Arithmetic Solutions Comparison Linear Equation Solving Practice Variable Representation in Inequalities Inequality Solutions Graphical Description	
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vertical or horizontal number line. Interpret sums of rational numbers in real world situations. Apply previous understandings of subtraction to subtract two rational numbers is the absolute value of their differences and apply the concept in real world contexts. Apply properties of operations as strategies to add and subtract rational numbers.			Complex Fraction Unit Rate Computation Proportional Relationship Testing in Tables Proportional Relationship Testing with Graphs Constant of Proportionality Identification in Equations Constant of Proportionality Identification in Tables Signed Number Addition and Subtraction Practice Rational Number Sum Interpretation Rational Number Subtraction Application Properties of Rational Numbers Application	
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			Mentor Texts: <i>How to Draw Cool Stuff: A Drawing Guide for Teachers and Students</i> by Catherine V. Holmes	
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Modifications and Accommodations	
Tier II and III Strategies	Small group instruction (Tier II) ● Explicit instruction ● Schema-based instruction ● Use visuals ● Use manipulatives ● Use a multi-sensory approach ● Guided notes and/or scaffold outline for written assignments ● Introduce key vocabulary before lesson ● Lesson taught again using a differentiated approach Small group or Individual instruction (Tier III) ● Intensive instruction ● Practice fact retrieval ● Lesson taught again using a differentiated approach ● Allow answers to be given orally or dictated ● Use visuals / Anchor Charts
Students with IEPs and 504 Plans	<u>Universal Design for Learning and Access for Students with Disabilities</u> <u>Movematics (dyscalculia.org)</u> Follow all the IEP/504 plan modifications

	Allow extra time to complete assignments or tests Guided notes and/or scaffold outline for written assignments Work in a small group Solidify and refine concepts through repetition Allow answers to be given orally or dictated Use multi-sensory teaching approaches Utilize assistive technology and materials Use large print books, Braille, or books on CD (digital text) Text to speech/audio recorded selections Amplification system as needed Leveled texts according to ability Fine motor skill stations embedded in rotation as needed Modified or constrained spelling word lists Provide anchor charts with high frequency words and phonemic patterns Students might need guidance as to what actions happen during partner work time. Invite students to discuss what it looks like when they are working with their partner while exploring the connecting cubes. Generate a list of shared expectations for partner work. Invite students to plan a strategy before they begin working. If time allows, invite students to share their plan with a partner. Some students may benefit from feedback that emphasizes effort and time on task. For
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	example, moving counters in the squares of the 5-frame or filling the 5-frame with one counter in each square.
Multi-language Learners	<u>Mathematical Language Development and Access for English Learners</u> Work toward longer passages as skills in English increase Use visuals Introduce/highlight key vocabulary before lesson Teacher models reading aloud daily Provide peer tutoring Utilize translation supports including bilingual dictionary, translation device, assistive technology, and peers Guided notes and/or scaffold outline for written assignments Provide students with English Learner leveled readers Support recommended by the grade specific “can do” list outlined by WIDA, including Key Uses Edition and Descriptor Name Charts Create a visual display of what students want to do or make with the connecting cubes. As students share ideas, include a drawing or annotation to illustrate connections. Invite students to use gestures as they count.
Accelerating Students	<u>Accelerating Students Guide</u> Create an enhanced set of introductory activities (e.g. advance organizers, concept maps, concept puzzles)

	Provide options, alternatives and choices to differentiate and broaden the curriculum Organize and offer flexible small group learning activities Provide whole group enrichment explorations Teach cognitive and methodological skills Use center, stations, or contracts Organize integrated problem-solving simulations Propose interest-based extension activities Expose students to beyond level texts.
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(NJDOE, 2024)

Unit 8: Probability and Sampling

Unit Goal: In this unit, students work with geometric and symbolic representations of square and cube roots. They understand and use notation such as 2 and 35 for square and cube roots. They understand the terms “rational number” and “irrational number,” using long division to express fractions as decimals. They use their understanding of fractions to plot rational numbers on the number line and their understanding of approximation of irrationals by rationals to approximate the number-line location of a given irrational. Students learn (without proof) that 2 is irrational. They understand two proofs of the Pythagorean Theorem—an algebraic proof that involves manipulation of two expressions for the same area and a geometric proof that involves decomposing and rearranging two squares. They use the Pythagorean Theorem in two and three dimensions, e.g., to determine lengths of diagonals of rectangles and right rectangular prisms, and to estimate distances between points in the coordinate plane.

Duration: 4 weeks

Essential Questions:

- How do geometric and symbolic representations aid in understanding square and cube roots, and what are their practical applications?

- What is the significance of notation such as $\sqrt{2}$ and $\sqrt[3]{5}$, and how does it enhance mathematical communication?
- What distinguishes rational numbers from irrational numbers, and how can long division be used to express fractions as decimals?
- How can rational numbers be plotted on the number line, and how can irrational numbers be approximated using rational numbers?
- Why is the irrationality of $\sqrt{2}$ important to understand, and how does it relate to other irrational numbers?
- What insights do algebraic and geometric proofs of the Pythagorean Theorem offer, and how do they contribute to understanding its applications?
- In what situations can the Pythagorean Theorem be applied in two and three dimensions, and how does it simplify geometric calculations and problem-solving?

Enduring Understandings:

- Geometric and symbolic representations of square and cube roots allow for understanding and manipulation of these mathematical operations.
- Notation such as $\sqrt{2}$ and $\sqrt[3]{5}$ represents square and cube roots respectively, enabling precise communication of mathematical concepts.
- Rational numbers can be expressed as fractions and decimals, while irrational numbers cannot be expressed as fractions and have non-repeating, non-terminating decimal expansions.
- Long division provides a method for expressing fractions as decimals, facilitating the conversion between different representations of rational numbers.
- Rational numbers can be plotted on the number line, while irrational numbers can be approximated by rational numbers to determine their approximate location on the number line.
- The irrationality of $\sqrt{2}$ is understood without formal proof, contributing to a foundational understanding of irrational numbers.
- Two proofs of the Pythagorean Theorem—algebraic and geometric—provide different perspectives on the relationship between the sides of a right triangle and inspire deeper mathematical reasoning.
- The Pythagorean Theorem has applications in two and three dimensions, allowing for calculations of lengths, distances, and areas in geometric contexts.

Key NJSLS Strands:

- **7.SP.A** Use random sampling to draw inferences about a population
 - **7.SP.A.1** Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.
 - **7.SP.A.2** Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.
- **7.SP.B** Draw informal comparative inferences about two populations.
 - **7.SP.B.3** Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability.
 - **7.SP.B.4** Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.
- **7.SP.C** Investigate chance processes and develop, use, & evaluate probability models.
 - **7.SP.C.5** Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around $\frac{1}{2}$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.
 - **7.SP.C.6** Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.
 - **7.SP.C.7** Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.
 - **7.SP.C.7.a** Develop a uniform probability model by assigning equal probability to all outcomes and use the model to determine probabilities of events.
 - **7.SP.C.7.b** Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.
 - **7.SP.C.8** Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.
 - **7.SP.C.8.a** Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

- **7.SP.C.8.b** Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.
 - **7.SP.C.8.c** Design and use a simulation to generate frequencies for compound events.
- **7.RP.A** Analyze proportional relationships and use them to solve real-world and mathematical problems.
- **7.NS.A** Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers
 - **7.NS.A.2** Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.
 - **7.NS.A.2.d** Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

Interdisciplinary Connections

- **NJSLSA.R7** Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words.
- **NJSLSA.SL1** Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others’ ideas and expressing their own clearly and persuasively.
- **NJSLSA.SL4** Present information, findings, and supporting evidence such that listeners can follow the line of reasoning and the organization, development, and style are appropriate to task, purpose, and audience.
- **MS-LS2-2** In this statistics unit students construct an explanation that predicts patterns of interactions among organisms.

21st-Century Skills

- **9.1.8.FP.2** Evaluate the role of emotions, attitudes, and behavior (rational and irrational) in making financial decisions.

Career Ready Practices

- **CRP2** Apply appropriate academic and technical skills.
- **CRP8** Utilize critical thinking to make sense of problems and persevere in solving them.

- **CRP11** Use technology to enhance productivity.

Technology Integration

- **9.4.8.TL.6** Collaborate to develop and publish work that provides perspectives on a real-world problem.

Skills	Instructional Topics	Assessment	Resources	Vocabulary
Statistics is used to gain information about a population by examining a sample of the population. Generalizations about a population from a sample are valid only if the sample is representative of that population. Random sampling tends to produce representative samples of the population and support valid inferences. Use data from a random sample to make inferences about a population with an unknown characteristic.	Probabilities of Single Step Events Chance Experiments What Are Probabilities? Estimating Probabilities Through Repeated Experiments More Estimating Probabilities Estimating Probabilities Using Simulation Probabilities of Multi-step Events Simulating Multi-step Experiments Keeping Track of All Possible Outcomes	Formative Assessments: Diagnostic Assessment Mid-topic Checkpoint Lesson Quizzes Lesson Self-assessment Teacher Observations Class Discussions Classwork/ Homework Exit Slips/Google Form Summative Assessments: Topic/Unit Assessments Constructed Response Questions	Materials: blank paper coins colored pencils compasses cylindrical household items empty toilet paper roll four-function calculators fruits or vegetables tracing paper graph paper	more likely less likely event chance experiment outcome equally likely as not likely unlikely impossible certain probability random sample space

Generate multiple samples, or simulated samples, of the same size to gauge variation in estimates or predictions. Informally gauge the extent of visual overlap between two numerical distributions with similar variabilities, measure the difference between the centers and express the difference as a multiple of the measure of variability. Draw informal comparative inferences about two populations by using the measures of center (mean and median) and measures of variability (interquartile range and mean absolute deviation) from random samples. Explain the probability of a chance event is a number between 0 and 1	Multi-step Experiments Designing Simulations Sampling Comparing Groups Larger Populations What Makes a Good Sample? Sampling in a Fair Way Using Samples Estimating Population Measures of Center Estimating Population Proportions More about Sampling Variability Comparing Populations Using Samples	Projects Cumulative Tests Performance Assessments Alternate and Benchmark Assessments: Progress Monitoring Assessments Cumulative/Benchmark Assessments	colored pencils scissors index cards rulers protractors glue or glue sticks graph paper grocery store circulars index cards internet-enabled device knife maps or satellite images of the school grounds markers materials assembled from the blackline master measuring cup	outcome simulation probability random event simulation tree (diagram) sample space tree (diagram) mean absolute deviation (MAD) distribution very different overlap mean median dot plot population
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that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around $\frac{1}{2}$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event. Approximate the probability of a chance event by collecting data on the chance process that it produces observing long run relative frequency. Predict the approximate relative frequency. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. Develop a probability			measuring spoons measuring tapes measuring tools metal paper fasteners (brass brads) meter sticks metric and customary unit conversion charts mixing containers number cubes pain paper bags paper clips paper cups paper plates pattern blocks pre-assembled polyhedra	sample survey mean absolute deviation (MAD) representative sample measure of center distribution center (of a distribution) spread random sample interquartile range (IQR) measure of variability box plot population sample random sample symmetric
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model, which may not be uniform, by observing frequencies in data generated from a chance process. Compare probabilities from a model to observed frequencies and explain possible sources of the discrepancy if the agreement is not good. Explain the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. Represent the sample space for a compound event using various methods such as, organized lists, tables, and tree diagrams. Identify the outcomes in the sample space which compose an event that has been described in everyday language.			protractors receipt tape recipes rulers rulers marked with centimeters rulers marked with inches scissors small disposable cups snap cubes sticky notes stopwatches straightedges straws string tape	proportion representative sample interquartile range (IQR) measure of variability meaningful difference overlap measure of center meaningful difference
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Design and use a simulation to generate frequencies for compound events.			tools for creating a visual display trundle wheels water yardsticks Activities: Square and Cube Root Exploration Notation Practice Rational vs. Irrational Number Sort Number Line Plotting Understanding the Irrationality of $\sqrt{2}$ Pythagorean Theorem Proofs Comparison Pythagorean Theorem Applications Mentor Texts: <i>Chasing Vermeer</i> by Blue Balliett	
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Modifications and Accommodations	
Tier II and III Strategies	Small group instruction (Tier II) ● Explicit instruction ● Schema-based instruction ● Use visuals ● Use manipulatives ● Use a multi-sensory approach ● Guided notes and/or scaffold outline for written assignments ● Introduce key vocabulary before lesson ● Lesson taught again using a differentiated approach Small group or Individual instruction (Tier III) ● Intensive instruction ● Practice fact retrieval ● Lesson taught again using a differentiated approach ● Allow answers to be given orally or dictated ● Use visuals / Anchor Charts
Students with IEPs and 504 Plans	<u>Universal Design for Learning and Access for Students with Disabilities</u> <u>Movematics (dyscalculia.org)</u> Follow all the IEP/504 plan modifications Allow extra time to complete assignments or tests Guided notes and/or scaffold outline for written assignments Work in a small group Solidify and refine concepts through repetition

	Allow answers to be given orally or dictated Use multi-sensory teaching approaches Utilize assistive technology and materials Use large print books, Braille, or books on CD (digital text) Text to speech/audio recorded selections Amplification system as needed Leveled texts according to ability Fine motor skill stations embedded in rotation as needed Modified or constrained spelling word lists Provide anchor charts with high frequency words and phonemic patterns Students might need guidance as to what actions happen during partner work time. Invite students to discuss what it looks like when they are working with their partner while exploring the connecting cubes. Generate a list of shared expectations for partner work. Invite students to plan a strategy before they begin working. If time allows, invite students to share their plan with a partner. Some students may benefit from feedback that emphasizes effort and time on task. For example, moving counters in the squares of the 5-frame or filling the 5-frame with one counter in each square.
Multi-language Learners	<u>Mathematical Language Development and Access for English Learners</u> Work toward longer passages as skills in English increase

	Use visuals Introduce/highlight key vocabulary before lesson Teacher models reading aloud daily Provide peer tutoring Utilize translation supports including bilingual dictionary, translation device, assistive technology, and peers Guided notes and/or scaffold outline for written assignments Provide students with English Learner leveled readers Support recommended by the grade specific “can do” list outlined by WIDA, including Key Uses Edition and Descriptor Name Charts Create a visual display of what students want to do or make with the connecting cubes. As students share ideas, include a drawing or annotation to illustrate connections. Invite students to use gestures as they count.
Accelerating Students	<u>Accelerating Students Guide</u> Create an enhanced set of introductory activities (e.g. advance organizers, concept maps, concept puzzles) Provide options, alternatives and choices to differentiate and broaden the curriculum Organize and offer flexible small group learning activities Provide whole group enrichment explorations

	Teach cognitive and methodological skills Use center, stations, or contracts Organize integrated problem-solving simulations Propose interest-based extension activities Expose students to beyond level texts.
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(NJDOE, 2024)

Unit 9: Integrating Math

Unit Goal: In this unit, students use concepts and skills from previous units to solve three groups of problems. In calculating or estimating quantities associated with running a restaurant, e.g., number of calories in one serving of a recipe, expected number of customers served per day, or floor space, they use their knowledge of proportional relationships, interpreting survey findings, and scale drawings. In estimating quantities such as age in hours and minutes or number of times their hearts have beaten, they use measurement conversions and consider accuracy of their estimates. Estimation of area and volume measurements from length measurements introduces considerations of measurement error. In designing a five-kilometer race course for their school, students use their knowledge of measurement and scale drawing. They select appropriate tools and methods for measuring their school campus, build a trundle wheel and use it to make measurements, make a scale drawing of the course on a map or a satellite image of the school grounds, and describe the number of laps, start, and finish of the race.

Duration: 2 weeks

Essential Questions:

- How can proportional relationships, survey interpretation, and scale drawings be used to solve problems related to running a restaurant or designing a race course?
- Why is it important to consider the accuracy and precision of measurement conversions when estimating quantities, such as age in hours or the number of servings in a recipe?
- What factors influence the measurement error when estimating area and volume from length measurements, and how can this error be minimized?

- What tools and methods are suitable for measuring a school campus and creating a scale drawing of a race course, and how do these choices affect the accuracy of the final design?
- How do considerations such as the number of laps, start, and finish of a race impact the design and layout of a race course, and how can measurement concepts be applied to address these factors effectively?

Enduring Understandings:

- Proportional relationships, survey interpretation, and scale drawings are essential tools for solving real-world problems, such as those encountered in running a restaurant or designing a race course.
- Measurement conversions play a crucial role in estimating quantities accurately, and students must consider the precision and reliability of their estimates.
- Estimation of area and volume measurements from length measurements requires an understanding of measurement error and its implications for the accuracy of calculations.
- Designing a race course involves applying measurement concepts and scale drawing skills to create a course that meets specific criteria, such as distance and layout, within the constraints of the school campus.

Key NJSLS Strands:

- **7.RP.A** Analyze proportional relationships and use them to solve real-world and mathematical problems
 - **7.RP.A.1** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.
 - **7.RP.A.2** Recognize and represent proportional relationships between quantities.
 - **7.RP.A.3** Use proportional relationships to solve multistep ratio and percent problems.
- **7.SP.B** Draw informal comparative inferences about two populations.
 - **7.SP.B.4** Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.
- **7.G.A** Draw, construct, and describe geometrical figures and describe the relationships between them.
 - **7.G.A.1** Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
- **7.G.B** Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.
 - **7.G.B.4** Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

- **7.G.B.6** Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
- **7.NS.A** Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.
 - **7.NS.A.2** Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.
 - **7.NS.A.2.d** Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.
 - **7.NS.A.3** Solve real-world and mathematical problems involving the four operations with rational numbers.
- **7.EE.A** Use properties of operations to generate equivalent expressions.
 - **7.EE.A.1** Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
- **7.EE.B** Solve real-life and mathematical problems using numerical and algebraic expressions and equations.
 - **7.EE.B.4** Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
 - **7.EE.B.4.a** Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p , q , and r are specific rational numbers. Solve equations of these forms with accuracy and efficiency. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.

Interdisciplinary Connections

- **NJSLSA.R7** Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words.
- **NJSLSA.SL1** Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively
- **NJSLSA.SL4** Present information, findings, and supporting evidence such that listeners can follow the line of reasoning and the organization, development, and style are appropriate to task, purpose, and audience.
- **MS-ETS1-1:** Students estimate sums and differences while defining the
- criteria and constraints of a design problem with sufficient precision to ensure a

● successful solution, taking into account relevant scientific principles.
21st-Century Skills
● 9.1.8.CR.2 Compare various ways to give back through strengths, passions, goals, and other personal factors.
Career Ready Practices
● CRP2 Apply appropriate academic and technical skills. CRP4: Communicate clearly and effectively with reason. CRP6: Demonstrate creativity and innovation.
Technology Integration
● 9.4.5.CT.3 Describe how digital tools and technology may be used to solve problems.

Skills	Instructional Topics	Assessment	Resources	Vocabulary
Compute unit rates involving ratios of fractions (complex fractions) in quantities measured in like or different units. Decide whether two quantities show a proportional relationship by testing for equivalent ratios in a table. Decide whether two	Running a Restaurant Planning Recipes Costs of Running a Restaurant More Costs of Running a Restaurant Restaurant Floor Plan Making Connections How Crowded Is this	Formative Assessments: Diagnostic Assessment Mid-topic Checkpoint Lesson Quizzes Lesson Self-assessment Teacher Observations Class Discussions Classwork/ Homework	Materials: blank paper coins colored pencils compasses cylindrical household items empty toilet paper roll	serving spreadsheet cell formula profit expense population density

quantities show a proportional relationship by graphing on a coordinate plane and observing whether the graph is a straight line through the origin. Identify the constant of proportionality (unit rate) in equations and verbal descriptions of proportional relationships. Identify the constant of proportionality (unit rate) in tables, graphs, and diagrams. Represent proportional relationships by equations using the constant of proportionality (unit rate). Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1,	Neighborhood? Fermi Problems More Expressions and Equations Measurement Error Designing a Course Measuring Long Distances Over Uneven Terrain Building a Trundle Wheel Using a Trundle Wheel to Measure Distances Designing a 5K Course	Exit Slips/Google Form Summative Assessments: Topic/Unit Assessments Constructed Response Questions Projects Cumulative Tests Performance Assessments Alternate and Benchmark Assessments: Progress Monitoring Assessments Cumulative/Benchmark Assessments	four-function calculators fruits or vegetables tracing paper graph paper colored pencils scissors index cards rulers protractors glue or glue sticks graph paper grocery store circulars index cards internet-enabled device knife maps or satellite images of the school grounds	percent error trundle wheel
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r) where r is the unit rate. Solve multistep ratio and percent problems using proportional relationships. Solve multistep ratio and percent problems using proportional relationships involving simple interest and sales tax. Solve multistep ratio and percent problems using proportional relationships involving markups and markdowns. Solve multistep ratio and percent problems using proportional relationships involving gratuities, commissions, and fees. Solve multistep ratio and percent problems using proportional relationships involving percent increase,			markers materials assembled from the blackline master measuring cup measuring spoons measuring tapes measuring tools metal paper fasteners (brass brads) meter sticks metric and customary unit conversion charts mixing containers number cubes pain paper bags paper clips	
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percent decrease, and percent error. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. Know the formulas for area and circumference of a circle. Solve problems using the formula for circumference of a circle and for the area of a circle. Informally derive the relationship between the circumference and area of a circle. Solve real-world and mathematical problems involving volume and surface area of three-dimensional			paper cups paper plates pattern blocks pre-assembled polyhedra protractors receipt tape recipes rulers rulers marked with centimeters rulers marked with inches scissors small disposable cups snap cubes sticky notes stopwatches	
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objects composed of cubes and right prisms. Apply the properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. Solve word problems by reasoning about their quantities and constructing simple equations of the form $p(x + q) = r$, where p, q, and r are specific rational numbers. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. Solve equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers with accuracy and efficiency.			straightedges straws string tape tools for creating a visual display trundle wheels water yardsticks Activities: Proportional Relationship Exploration Survey Interpretation Practice Scale Drawing Exercise Measurement Conversion Challenge	
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			Estimation of Area and Volume Trundle Wheel Construction Race Course Design Project Recipe Calorie Calculation Customer Traffic Prediction Precision and Accuracy Analysis Measurement Error Investigation Campus Measurement Scavenger Hunt Map Scale Analysis Race Course Model Building Lap Counting and Timing Exercise Mentor Texts:	
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			<i>Eat Your Math Homework: Recipes for Hungry Minds</i> by Ann McCallum <i>The Great Kapok Tree: A Tale of the Amazon Rain Forest</i> by Lynne Cherry <i>The Wright Brothers: How They Invented the Airplane</i> by Russell Freedman	
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	● Use visuals / Anchor Charts
Students with IEPs and 504 Plans	<u>Universal Design for Learning and Access for Students with Disabilities</u> <u>Movematics (dyscalculia.org)</u> Follow all the IEP/504 plan modifications Allow extra time to complete assignments or tests Guided notes and/or scaffold outline for written assignments Work in a small group Solidify and refine concepts through repetition Allow answers to be given orally or dictated Use multi-sensory teaching approaches Utilize assistive technology and materials Use large print books, Braille, or books on CD (digital text) Text to speech/audio recorded selections Amplification system as needed Leveled texts according to ability Fine motor skill stations embedded in rotation as needed Modified or constrained spelling word lists Provide anchor charts with high frequency words and phonemic patterns Students might need guidance as to what actions happen during partner work time. Invite

	students to discuss what it looks like when they are working with their partner while exploring the connecting cubes. Generate a list of shared expectations for partner work. Invite students to plan a strategy before they begin working. If time allows, invite students to share their plan with a partner. Some students may benefit from feedback that emphasizes effort and time on task. For example, moving counters in the squares of the 5-frame or filling the 5-frame with one counter in each square.
Multi-language Learners	<u>Mathematical Language Development and Access for English Learners</u> Work toward longer passages as skills in English increase Use visuals Introduce/highlight key vocabulary before lesson Teacher models reading aloud daily Provide peer tutoring Utilize translation supports including bilingual dictionary, translation device, assistive technology, and peers Guided notes and/or scaffold outline for written assignments Provide students with English Learner leveled readers Support recommended by the grade specific “can do” list outlined by WIDA, including Key Uses Edition and Descriptor Name Charts Create a visual display of what students want to do or make with the connecting cubes. As students share ideas, include a drawing or annotation to illustrate connections. Invite students to use gestures as they count.

Accelerating Students	<u>Accelerating Students Guide</u> Create an enhanced set of introductory activities (e.g. advance organizers, concept maps, concept puzzles) Provide options, alternatives and choices to differentiate and broaden the curriculum Organize and offer flexible small group learning activities Provide whole group enrichment explorations Teach cognitive and methodological skills Use center, stations, or contracts Organize integrated problem-solving simulations Propose interest-based extension activities Expose students to beyond level texts.
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(NJDOE, 2024)

Prepared by Julie Lang, MA

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